ON LOCAL PERTURBATIONS OF A BOUNDARY LAYER WITH SMALL SKIN FRICTION

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This paper studies the effect of two-dimensional surface irregularities on the flow in a plane steady boundary layer with small skin friction in an incompressible fluid. A detailed analysis is carried out for the flow regime with a given pressure gradient determined on the scale of the small irregularity by its shape. It is shown that there is a critical value of the height (depth) of the irregularity at which the skin friction first becomes zero, and the nonuniqueness of the corresponding solution is established.

The first studies of the effect of small two-dimensional surface irregularities on boundary layer flow were carried out in [1, 2]. It was found (see also [3, 4]) that on the irregularity scale three characteristic regimes are possible. Flow with interaction is observed when the pressure gradient and the displacement effect of the boundary layer are interdependent, while a decrease or increase in the longitudinal dimension of the irregularity as compared with the interaction region scale leads to the regime without a displacement effect (compensation regime) or flow with a given pressure gradient, respectively.

In all these studies the original unperturbed boundary layer has a finite (nondimensional) skin friction. The present study is concerned with the effect of two-dimensional surface irregularities on the incompressible fluid flow in a boundary layer with a small value of the skin friction. It is of interest because of the possibility of analyzing the characteristic properties of the boundary layer and, in particular, the especially important question of the uniqueness of the solutions.

1. The steady flow in a two-dimensional Prandtl boundary layer developing along the surface of a body in a high-Reynolds-number flow is described by a solution of the following boundary value problem:

\[
\frac{\partial \Psi}{\partial Y} \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Psi}{\partial Y} + \frac{\partial p_s}{\partial x} = \frac{\partial \Psi}{\partial Y} \\
Y=0: \quad \Psi = \frac{\partial \Psi}{\partial Y} = 0, \quad Y = \infty: \quad \frac{\partial \Psi}{\partial Y} = -u_s(x) \\
x=x_0: \quad \Psi = \Psi^*(Y), \quad \frac{dp_s}{dx} = -u_s \frac{du_s}{dx} \\
u_s(1) = 1, \quad x_0 < x < 1, \quad x_0 > 0 \\
Y=Re^{1/2}Y, \quad \Psi = Re^{1/2} \Psi
\]

Here and below, \(LU_0^\Psi\) is the stream function; \(u = \Psi_x, v = -\Psi_y\) are the nondimensional velocity components along the axes \(Lx\) and \(Ly\) of the Cartesian coordinate system fitted to the body surface \(y=0\); \(P_0 + \rho U_0^2 p\) is the pressure, and \(\rho\) is the fluid density. The characteristic scale \(L\) is the distance from the origin to the point on the body surface in whose vicinity the irregularity is located (Fig. 1) and the characteristic velocity \(U_0\) and pressure \(P_0\) are their values at this point.

In (1.1) the functions \(u_s(x)\) and \(p_s(x)\) determine the velocity and pressure distributions at the outer edge of the boundary layer and are assumed to be known from the solution of the problem of the external potential flow, while \(\Psi^*(Y)\) determines the initial velocity profile; \(Re=U_0L/v\) is the Reynolds number, where \(v\) is the kinematic viscosity coefficient.

We will use the following property of the boundary layer equations. If \(\Psi(x, Y)\) is a solution of the problem (1.1), then to this solution it is possible to add an eigenfunction of the form \(\sigma \Psi(x) \partial \Psi/\partial Y\), where \(\sigma\) is a small parameter and \(\gamma(x)\) is arbitrary.
The entire analysis which follows is concerned with the study of a boundary layer with small skin friction. Therefore, we will take as a solution of the problem (1.1) the well-known similarity solution of Falkner and Skan [6] (see also [5, 7]), which has the form:

\[ u_s = x^m, \quad p_s' = -mx^{2m-1} \]

\[ \Psi = x^{(m+1)/2} f(\eta), \quad \eta = Yx^{(1-m)/2}, \quad \Psi' = \Psi(x, y) \]

and gives zero skin friction: \( f''(0) = 0 \) at \( m = -\lambda_0 = -0.0904 \) [6, 8, 9]. Using the property pointed out above, to this solution we add the eigenfunction [10, 11] determining the distribution of the surface friction \( \tau_w \). As a result, for the unperturbed boundary layer we have

\[ Y = x + o\delta x f'(n) + o(o) \]

\[ p = (1 - x^{-2\lambda_0})/2, \quad \tau_w = \left[ \frac{\partial^2 \Psi}{\partial Y^2} \right]_{x=0} = \sigma b_0 x f''(0) + O(o) \]

where \( f(\eta) \) is the solution of (1.2) for \( m = -\lambda_0 \) and \( \sigma \) is a small parameter: \( \sigma = \sigma(Re) \to 0 \) as \( Re \to \infty \) and \( b_0 \) and \( k \) are arbitrary constants.

Therefore, the unperturbed boundary layer is under the action of an adverse pressure gradient and has a small value of the skin friction which is positive for \( b_0 > 0 \) while for \( b_0 < 0 \), there is a slow reversed flow near the body surface. Note that an analysis of the effect of small surface irregularities on the boundary layer with small surface friction will make it possible to clarify many of properties associated with flow separation.

2. Let us assume that at the distance \( L \) (\( x=1 \)) from the origin there is a surface irregularity (Fig. 1) with longitudinal and transverse dimensions of the order of \( e(Re) \to 0 \) and \( h(Re) \to 0 \) as \( Re \to \infty \). We will study the flow structure near this irregularity. From (1.2) and (1.3) we have:

\[ |s| \to 0: \quad u_s = 1 - \lambda_0 s + O(s^2), \quad p_s = \lambda_0 s + O(s^2), \quad s = x - 1 \]

\[ s \to 0: \quad \Psi = \Psi_0(Y) + O(|s|) + \sigma [\Psi_1(Y) + O(|s|)] + O(o) \]

\[ \Psi_0 = f(Y), \quad \Psi_1 = b_0 f'(Y) \]

\[ Y \to 0: \quad f = \frac{-\lambda_0}{6} Y^3 + O(Y^5), \quad f'(\infty) = 1 \]