ON THE UNIVERSAL AERODYNAMIC RELATIONS FOLLOWING FROM THE NEWTONIAN THEORY

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A new, more general than that of [1], approach to the analysis of aerodynamic coefficients within the framework of the modified Newtonian theory is developed. Universal aerodynamic relations, similar to those of [1] but easily solvable with respect to the aerodynamic coefficients, are derived. Universal relations for the areas of the "windward" zone projections onto planes normal to the axes of a Cartesian reference frame are obtained.

It was shown in [1] that within the framework of the modified Newtonian theory [1, 2] the aerodynamic force coefficients are related by universal (i.e., independent of the body shape) differential equations with such parameters as the areas of the projections of the "windward" body surface. If the velocity vector direction with respect to a certain Cartesian body-fitted reference frame $X^1Z$ is determined by the angles $\alpha$ and $\varphi$, as shown in Fig. 1, then these equations have the form:

$$12C_{za} + \cot \alpha (C_{ze})' + (C_{ze})'' + (C_{ze})''\sin^{-2}\alpha = 6K^*S_a$$

or

$$6C + \cot \alpha (C)'_z + (C)''_z + (C)''_z\sin^{-2}\alpha = 2K^*S_s$$

$$K_s = K/S^*, \quad C = (C_{ze} - C_{ze}C_z), \quad S_s = (S_{x}, S_{y}, S_{z})$$

$$S_x = \int n_x dS, \quad S_y = \int n_y dS, \quad S_z = \int n_z dS$$

$$S_s = \cos \alpha S_x - \sin \alpha \cos \varphi S_y + \sin \alpha \sin \varphi S_z$$

(0.1)

Here, $C_{za}$ are the aerodynamic force coefficients in the body-fitted reference frame, $C_{za}$ is the drag coefficient in the velocity reference frame, $K$ is a constant coefficient (in the classical case, $K=2$), $S$ is a scale area, $S_s$ is the area of the projection of the "windward" body surface onto a plane perpendicular to the freestream, $S_{xy}$ are the areas of the projections of the "windward" body surface onto planes perpendicular to the axes of the reference frame, $n_{xyz}$ are the components of the unit normal $n$ to the body surface, and $S$ is the "windward" body surface.

Various methods of aerodynamic calculation, in which the aerodynamic coefficients are calculated from these universal relations, have been developed [3--8].

However, the aerodynamic coefficients cannot be expressed by means of these formulas in a sufficiently simple, convenient-to-use form. Moreover, the problem of solving these equations in terms of the aerodynamic coefficients has developed into an independent, rather complicated problem.

At the same time, within the framework of the modified Newtonian theory another approach may exist which makes it possible to derive relations similar to those mentioned above but free from their shortcomings.

1. Within the framework of the modified Newtonian theory, the aerodynamic force coefficients in a body-fitted reference frame can be written in the form:

$$C = K^* \sum_{j=0}^{2} (-1)^j C_{a}^j \cos^2 \alpha \sin^j \alpha \sum_{m=0}^{j} (-1)^m C_{m}^j \cos^m \varphi \sin^m \varphi \Phi$$

$$\Phi = (F_{m-r, r}, F_{m-1, 2-k}, F_{m-k, k})$$

$$F_{k,k} = \int n_x^k n_y^{k-1-k} n_z^k dS$$

where $C_{a}^j$ and $C_{m}^j$ are the binomial coefficients.
It can be shown that in the Newtonian flow past a body, the $F_{i,k}$ parameters are interrelated by universal, i.e., independent of the body shape, differential equations which can be written in the following generalized form:

$$ (F_{i,k})_{1}^{'} = \frac{\sin \alpha}{\cos \alpha} \sum_{j=0}^{k} \left( -1 \right)^{j} C_{2}^{j} \cos^{2-j} \theta \sin^{j} \theta \left( F_{i,k} \right)_{1}^{'} \; ; \; \; \xi = (\alpha, \varphi) \tag{1.2} $$

2. By performing appropriate operations on Eq. (1.1), we can derive, in view of universal relations (1.2), new universal differential equations relating the coefficients $C_{x,z}$ and parameters $F_{i,k}$

$$ [\tan^{n-p-1} \alpha \sin^{2} \alpha [\left( \cos^{2-n} \alpha \sin^{n} \alpha \right)^{-1} C_{x,z}]^{''} = $$

$$ K^{*} (-1)^{j} C_{2}^{j} (j-n)(j-p) \tan^{n-p-1} \alpha \left( \cos^{2} \alpha \right)^{-1} \sum_{m=0}^{j} (-1)^{m} C_{m}^{m} \cos^{2-m} \varphi \sin^{m} \varphi \tag{2.1} $$

where $j = 3 - n - p, n \neq p$, and $(n, p) = 0, 1, 2$.

In particular, at $j = 0$ from Eq. (2.1) we obtain

$$ C - \frac{1}{2} \sin 2 \alpha (C_{x,z})^{'} + \frac{1}{2} \sin^{2} \alpha \left( C_{x,z}^{''} - K^{*} \Phi_{0} \right) \tag{2.2} $$

$$ \Phi_{0} = (F_{0,0}, F_{0,1}, F_{1,2}) $$

The left-hand side of Eq. (2.2) can be written down in the following two contracted forms:

$$ K^{*} \Phi_{0} = \frac{1}{2} \sin^{2} \alpha \left[ \sin^{2} \alpha \left( \left( \sin^{2} \alpha \right)^{-1} C_{x,z} \left( \left( \sin^{2} \alpha \right)^{-1} C_{x,z} \right) \right) \right] \tag{2.2} $$

Equations (2.2) are the counterparts of Eqs. (0.2) derived within the framework of the approach [1]; however, as distinct from the latter, these newly derived equations can be directly solved by double integration of their curtailed form with respect to the angle $\alpha$; as for the angle $\varphi$, its influence manifests itself only indirectly, via the "windward" surface boundary.

3. In a similar fashion, the following expressions for the drag coefficient $C_{d}$ in the velocity reference frame can be derived