STABILITY OF A SURFACE-CHARGED VISCOUS JET IN AN ELECTRIC FIELD  

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The stability of a surface-charged cylindrical jet in a longitudinal uniform electric field with respect to capillary perturbations is investigated in the linear approximation. The evolution of both axisymmetric and azimuthal-periodic perturbations is analyzed. In the latter case the first two modes among the azimuthal wavenumbers - bending and Bohr - are considered. Axisymmetric and bending instabilities lead to the transverse disintegration of the jet into individual drops and the Bohr mode to the longitudinal separation of the input jet into two parts. It is found that the axisymmetric and bending instabilities, respectively, can be completely suppressed and significantly attenuated by means of an external longitudinal field. In this case the role of the Bohr mode becomes more important leading under certain conditions to longitudinal longwave jet splitting. Events which can be interpreted as manifestations of longitudinal partition of the jet (dumbbell-like cross-section, branching nodes) are observed in experiments with evaporating polymer-solution microjets.

The possibility of completely stabilizing viscous jets with respect to capillary instability using the electric force effect is studied. Volume-charged and uncharged dielectric jets in a longitudinal electric field and a charged ideal conductor are not completely stable with respect to axisymmetric perturbations [1--3]. In the case of a volume-charged dielectric in a longitudinal field the maximum growth rate is displaced in the short and long-wave directions, respectively, as the charge and the field strength increase. Although with increase in the strength the maximum growth rate decreases, for no values of the problem parameters is perturbation damping observed at all wavelengths [1]. For conducting and dielectric jets there always exist domains of instability with respect to the axisymmetric mode which are located, respectively, on the shortwave and longwave intervals of the perturbation spectrum. In the case of a surface-charged conductor, as distinct from a volume-charged dielectric, the longwave perturbations are strongly damped [2--3]. These data suggest that the conditions for suppressing instabilities over the entire perturbation spectrum can be created under the joint influence of the dielectric properties and the interaction between surface charges.

1. FORMULATION OF THE PROBLEM

We will consider a surface-charged fluid flow with a free boundary in an external electric field. The analysis will be carried out in the frozen-in charge approximation [4, 5].

Since there is no volume charge, the electric field acts on the jet via the boundary and in the cylindrical coordinates $r, \theta, z$ the system of equations has the form:

\begin{align*}
\frac{\partial \nu}{\partial t} + (\nu \nabla) \nu - \frac{w^2}{r} &= -\frac{1}{\rho} \frac{\partial \rho}{\partial r} + \nu \left( \frac{\Delta \nu}{r^2} - \frac{2}{r^2} \frac{\partial \nu}{\partial \theta} \right), \\
\frac{\partial w}{\partial t} + (\nu \nabla) w - \frac{w v}{r} &= -\frac{1}{\rho r} \frac{\partial \rho}{\partial \theta} + \nu \left( \frac{\Delta w}{r^2} + \frac{2}{r^2} \frac{\partial w}{\partial \theta} \right), \\
\frac{\partial u}{\partial t} + (\nu \nabla) u &= -\frac{1}{\rho} \frac{\partial \rho}{\partial z} + \nu \frac{\partial u}{\partial z}, \\
\frac{\partial w}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r v) + \frac{1}{r} \frac{\partial w}{\partial \theta} &= 0,
\end{align*}

\begin{align*}
\Delta \varphi &= 0, \\
(\nu \nabla) \Phi &= v \frac{\partial \Phi}{\partial r} + \frac{w \partial \Phi}{r} + \frac{u \partial \Phi}{\partial \theta}, \\
\Delta \Phi &= \frac{1}{r} \frac{\partial}{\partial r} (r \Phi) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}.
\end{align*}
Here, $u$, $v$, and $w$ are the axial, radial, and azimuthal velocities, $\phi$ is the electric field potential, $\rho$ is density, $v$ is the kinematic viscosity, and $\Phi$ is an arbitrary function. On the free boundary $r = f(t, z, \theta)$ we impose the conditions

$$v = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial z} + w \frac{\partial f}{\partial \theta}$$

(1.2)

$$\frac{\partial}{\partial t}(\sigma f) + \frac{\partial}{\partial z}(\sigma f u) + \frac{\partial}{\partial \theta}(\sigma w) = 0$$

(1.3)

$$\phi^i = \phi^f, \quad D_n^i - D_n^f = \sigma$$

(1.4)

$$P_n = F \cdot E - TK \kappa, \quad K = \frac{1}{f} - f z'' - \frac{f u''}{f^2}$$

(1.5)

$$F = D_t^i E^f - D_t^f E^i - (D^i E^f - D^f E^i) a/2$$

(1.6)

Here and in what follows, terms proportional to $f^2$ have been omitted, $E = -\nabla \phi$ is the electric field strength, $D = \epsilon \epsilon_0 E$, $\epsilon$ and $\epsilon_0$ are dielectric constants, $\sigma$ is the surface charge density, $T$ is the surface tension, $p_n$ is the fluid stress tensor, $F_t$ is the electric force per unit area, $n$ is the outward normal to the jet, and the superscripts $e$ and $i$ relate to points outside and inside the fluid, respectively. Equation (1.2) serves to determine the unknown boundary, relation (1.3) describes the variation of the frozen-in charge along the free surface of the moving fluid.

2. STEADY-STATE AXISYMMETRIC FLOW

In the case of axisymmetric jet flow we have

$$u = \frac{1}{r} \psi, \quad v = -\frac{1}{r} \psi, \quad w = 0$$

(2.1)

The kinematic boundary conditions (1.2) and (1.3) can be written in the form:

$$\psi = Q/2\pi, \quad I = 2\pi f u \sigma = 2\sigma Q f, \quad Q_1 = \pi f^2 u$$

(2.2)

where equality (2.2) for the stream function $\psi$ is equivalent to the relation $f = v/u$, $I = \text{const}$ is the electric current transported by the jet, and $Q = \text{const}$ is the volume flow rate of fluid through an arbitrary transverse jet cross-section. We will assume that the fluid is located in a uniform electric field of strength $E_0$ parallel to the jet axis. The total electric field is the superposition of the external uniform field and that of the surface charges. In the case of a weakly varying jet profile $f(z) \ll 1$ the latter is similar to the field of a charged cylinder of constant radius. Then, neglecting the distortion of the axial external field due to the surface charge, we have $E_n^i = 0$, $E_n^f = \sigma / \epsilon_0$, and $E_t^i = E_t^f = E_0$, where the subscript $\tau$ denotes the direction tangential to the surface in the rz plane.

For the coordinate, velocities, and pressure we will use the following scales:

$$r_0, \quad Q^2/\pi, \quad \rho Q^2/\pi r_0^4$$

where $r_0 = f(z = 0)$ is the initial radius of the jet. In dimensionless variables with allowance for (2.1) the system (1.1) and the boundary conditions (1.2), (1.5) take the form:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} \right) \right)$$

(2.3)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}} \left( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{\partial v}{\partial z} \right) \right)$$

(2.4)

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