BIFURCATION REGIMES OF FREE CONVECTION IN THE PRESENCE OF THERMAL RADIATION AND CAVITY INCLINATION

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The bifurcation regimes of free convection in closed cavities with heating from below have been investigated numerically by many authors [1]. In the situations considered the equilibrium solution conditions were disturbed by only one factor, e.g. the inclination of the cavity to the vertical, the motion of one of the boundaries, a change in the equilibrium temperature distribution, etc. In this paper, the simultaneous influence of two factors that disturb the fluid equilibrium conditions, namely thermal radiation and a slight inclination of the cavity relative to the vertical, are investigated. It is shown that, for the simultaneous action of two destabilizing factors, a near-equilibrium solution is possible.

1. The problem a transparent gas in a square cavity with heating from below (Fig. 1) is described by the system of natural convection equations in the Boussinesq approximation [2]. On the boundaries, the velocity vector satisfies the no-slip conditions and the temperatures of the surfaces $S_0$ and $S_1$ are constant:

$$v_{|z}=0, \quad T_{|z}=T_0, \quad T_1=T_1 > T_0$$  \hspace{1cm} (1.1)

On the boundaries $S_2$ and $S_3$, we will consider the three variants of the boundary conditions for the temperature discussed below.

For taking account of the radiative heat transfer on the boundaries we used the "absolutely black" surface model. Such surfaces radiate and absorb heat but do not reflect it [3]. Assuming that the emissivities and the absorptivities of all the surfaces $S_m$ ($m=0, 1, 2, 3$) are identical, we can write the local density of the resulting radiation flux on an elementary area $S^{(m)}$ in the form:

$$q(s^{(m)})=\varepsilon e \left[ T^4(s^{(m)}) - \sum_{j=0}^{r} K(r^{(m)}, r^{(j)}) T^4(s^{(j)}) ds^{(j)} \right]$$

Here, $\sigma$ is the Stefan constant, $e$ is the emissivity, and the function $K(r^{(m)}, r^{(j)})$ depends on the location and orientation of the radiating surfaces.

Assuming that the surface $S_2$ is thermally insulated, we obtain the following boundary condition:

$$\frac{\partial T(S_2)}{\partial x} - \varepsilon e \left[ T^4(S_3) - T^4(S_2) - T^4(S_1) \right] = 0$$  \hspace{1cm} (1.2)

Here, $\lambda$ is the thermal conductivity of the gas in the cavity.

Using the cavity height $H$ as the characteristic length scale ($L=H$) and the nondimensional temperature $\theta=(T-T_0)/(T_1-T_0)$, we can represent the boundary condition (1.2) in the form:

$$\eta_0 \frac{\partial \theta(S_2)}{\partial x} - Rd \left[ 1 + \theta(S_2) \right]^4 - \int_{S_0} K(r_{1r}, r_{2r}) ds_0 - \int_{S_1} \left[ K(r_{1r}, r_{2r}) \right] ds_1 = 0$$

$$\eta_0^4 \int_{S_1} K(r_{1r}, r_{2r}) ds_1 - \int_{S_2} K(r_{2r}, r_{2s}) ds_2 = 0$$

where $Rd = \sigma e HT_0^3 / \lambda$.

After nondimensionalization and the introduction of the stream function $\psi$, and vorticity $\varphi$, the system of equations
Fig. 1. Geometry of the calculation domain.

takes the form:

\[ y = \frac{\partial \psi}{\partial y}, \quad x = -\frac{\partial \psi}{\partial x}, \quad \varphi = (\text{curl } V). \]

\[ \frac{\partial \varphi}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \varphi}{\partial y} = \Delta \varphi + G \left[ \frac{\partial T}{\partial x} \cos \alpha - \frac{\partial T}{\partial y} \sin \alpha \right] \]

\[ \Delta \psi + \varphi = 0 \]

\[ \frac{\partial T}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{1}{Pr} \Delta T \]

\[ G = g \beta (T_1 - T_0) H^3 / \nu^2, \quad Pr = \nu / \chi \]

Here, \( \alpha \) is the cavity inclination angle relative to the vertical, \( G \) is the Grashof number, and \( Pr \) is the Prandtl number.

The boundary conditions (1.1) for the stream function and the temperature on two of the surfaces are as follows:

\[ \psi|_{x=0} = 0, \quad \psi / \partial n|_{x=0} = 0, \quad T|_{S_1} = 0, \quad T|_{S_2} = 1 \] (1.5)

The solution of the problem (1.3)-(1.5) depends on five nondimensional parameters: the Grashof number \( G \), the Prandtl number \( Pr \), \( \eta_0 \), \( Rd \), and the cavity inclination angle \( \alpha \). Using these parameters, we can calculate the Rayleigh number \( Ra = G \cdot Pr \).

The problem was solved using a separate variant of the two-field method [1]. The evolutionary equations for the vorticity and the temperature were solved by the streamwise-crosswise sweep method using the parabolic-spline approximation with an accuracy \( O(h^2) \). The Poisson equation for the stream function was solved using the iterative streamwise-crosswise sweep method. The stream function was approximated by a discrete cubic spline which ensured an accuracy \( O(h^3) \). The vorticity on the boundaries was determined using the Woods formula with underrelaxation. This method is described in detail in [4]. Most calculations were performed for a fixed Prandtl number equal to 0.71 (air) and \( \eta_0 = 1.1 \).

In the main calculation series, we used a square grid with a 1/20 step.

2. A specific feature of the problem of heating from below in the absence of radiative heat transfer \( (Rd = 0) \) and cavity inclination \( (\alpha = 0) \) is the existence of an equilibrium solution [2]:

\[ \psi^0(x, y) = 0, \quad T^0(x, y) = 1 - y \]

This solution loses stability when the Rayleigh number exceeds the critical values \( Ra = Pr G_j (j = 1, 2, \ldots) \). For \( Ra > Ra_j \), the amplitude curves are described by a square-root dependence. In bifurcation theory, this corresponds to a singularity of the first kind, i.e., a Whitney fold.

The problem was solved for three variants of the boundary conditions on the vertical boundaries \( S_2 \) and \( S_3 \): (i) the boundaries are thermally insulated, (ii) the left boundary is thermally insulated while on the right boundary the temperature distribution is linear, and (iii) the temperature distribution is linear on both vertical boundaries:

\[ \frac{\partial T}{\partial x} = 0, \quad x = 0, \quad x = 1 \]