PARTICLE INTERACTION IN A FLOW WITH A PARABOLIC VELOCITY PROFILE

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The hydrodynamic interaction of two rigid spherical particles in a viscous incompressible fluid with the velocity at infinity represented by a second-degree polynomial in the coordinates is considered. An analytical solution of the problem is suggested. The forces and torques exerted on the particles and also the linear and angular particle velocities are calculated. The results are compared with previous theoretical and experimental data.

In [1, 2], an analytical method of solving the problem of hydrodynamic interaction between particles in a flow with the velocity represented by a polynomial of arbitrary integer order was suggested. The problem of particle interaction in a flow with a parabolic velocity profile is of interest because, as noted in [2], the particle behavior in flows with polynomial velocity profiles of even degree (for example, parabolic) should differ substantially from that in flows with polynomial velocity profiles of odd degree, for example, a linear profile.

1. FORMULATION OF THE PROBLEM

We will consider the hydrodynamic interaction of two rigid spherical particles A and B of the same radius a, immersed in an unbounded incompressible fluid with the viscosity \( \eta \). It is assumed that no external forces or torques are exerted on the particles and the particle size is small enough, for the Reynolds number to be small (\( \text{Re} < 1 \)). The flow velocity at infinity \( \mathbf{U} \) is a quadratic function of the coordinates:

\[
\mathbf{U}_i = E_{ij} x_j + \Omega_{ij} x_j = C_{ijkl} x_k x_l, \quad E_{ii} = \Omega_{ii} = C_{ik} = C_{ik} = 0
\]

The location of the centers of the spheres A and B relative to the flow is denoted by \( \mathbf{r}_a \) and \( \mathbf{r}_b \), respectively. The equations for the flow velocity \( \mathbf{u}(\mathbf{x}) \) and the pressure \( p(\mathbf{x}) \) are written in the Stokes approximation:

\[
\nabla \cdot \mathbf{u} = 0 \quad (1.1)
\]

\[
\eta \nabla^2 \mathbf{u} = \nabla p \quad (1.2)
\]

On the particle surface, the no-slip condition is valid:

\[
|\mathbf{X}_a| = a: \quad u_i + U_i(A) + E_{ij} X_{aj} + E E_{ij}(A) X_{aj} = \mathbf{V}_i^a + \Gamma_{ij} X_{aj}
\]

\[
|\mathbf{X}_b| = b: \quad u_i + U_i(B) + E_{ij} X_{bj} + E E_{ij}(B) X_{bj} = \mathbf{V}_i^b + \Gamma_{ij} X_{bj}
\]

At infinity, we have:

\[
|\mathbf{X}| \rightarrow \infty: \quad u_i \rightarrow 0 \quad (1.5)
\]

Here, we have introduced the following notation:
The vectors $V^a$, $V^b$, $\Gamma^a$, and $\Gamma^b$ are the absolute linear and angular velocities of the spheres $A$ and $B$. The vectors $U(A)$ and $U(B)$ are the flow velocities at points coinciding with the centers of the spheres $A$ and $B$, respectively. The linear and angular velocities of the spheres are unknown functions of the vector $r$ and the parameter $a/r$.

2. THE SOLUTION

As in the case considered in [1, 2], by virtue of the linearity we can represent the solution of the hydrodynamic equations (1.1)--(1.2) with the boundary conditions (1.3)--(1.5) as a sum of solutions of several problems. The first problem consists in finding a solution of the Stokes equations with the following boundary conditions on the surface of the two spheres:

\[
[X_a] = a: \quad u_i + E_{ij}X_{aj} + E E_{ij}(A)X_{ai} = 0 \\
[X_b] = a: \quad u_i + E_{ij}X_{bj} + E E_{ij}(A)X_{bi} = 0
\]

As before, at infinity we require the disturbances to be damped. In fact, these boundary conditions mean that the particles travel with the local linear and angular velocities of a certain undisturbed flow with a linear velocity at infinity (the $\alpha$ problem in [2]).

In the second problem, the Stokes equations are solved for the following boundary conditions on the surface of the two spheres:

\[
[X_a] = a: \quad u_i + C_{ij}X_{aj} = 0 \\
[X_b] = a: \quad u_i + C_{ij}X_{bj} = 0
\]

These conditions mean that the particles travel without velocity slip in the flow with a parabolic velocity profile.

The third problem consists in finding the solution of the hydrodynamic equations with the following boundary conditions:

\[
[X_a] = a: \quad u_i = 0 \\
[X_b] = a: \quad u_i + \Delta E E_{ij}X_{bi} = 0 \\
\Delta E E_{ij}(B) = (C_{ijk}r_k + C_{jkl}r_l)
\]

The fourth problem is that of the motion of particles with linear and angular velocities in a fluid at rest at infinity under the following boundary conditions:

\[
[X_a] = a: \quad u_i = U_i^a + \omega_i^b X_{aj} \\
[X_b] = a: \quad u_i = U_i^b + \omega_i^b X_{bj}
\]

Here, the relative linear and angular velocities of the spheres are introduced as follows:

\[
U_i^a = V_i^a - U_i(A), \quad \omega_i^a = \Gamma_i^a - \Omega_i^a - W_{ij}(A) \\
U_i^b = V_i^b - U_i(B), \quad \omega_i^b = \Gamma_i^b - \Omega_i^b - W_{ij}(B)
\]

The solution of the first and fourth problems was found in [2] (the $\alpha$ and $\beta$ problems). To obtain the solution of the third problem, we can also use the results of [1, 2]. The expressions for the flow velocity and pressure in the second problem are sought in the same form as in the $\alpha$ problem. The only difference is that the velocity function must now satisfy the symmetric transformation given in [2]:

\[
u(x) = u(-x + r) \quad (2.1)
\]