PROPERTIES OF HYPERSONIC SEPARATED FLOWS
AT MODERATE REYNOLDS NUMBERS

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The hypersonic rarefied flow past a flat plate with a transverse barrier and past a plate with a bend in the generator (a compression angle) is studied at Reynolds numbers \(\text{Re} \leq 10^4\). Direct statistical modeling (Monte Carlo method) is used to investigate the characteristics of the separated flow formed on the plate as a function of the Reynolds number, the surface temperature, the barrier dimensions, and the internal degrees of freedom of the molecules. The results obtained are compared with those for analogous high-Re flows. The possibility of using the similarity criteria derived for the continuum flow regime is considered.

Much work has been done on separated boundary layer flows, both laminar and turbulent, at high Reynolds numbers (\(\text{Re} \geq 10^5\)) (see, for example, [1, 2]). The first systematic experimental studies of various two-dimensional supersonic separated flows and attempts to theoretically account for their salient features were made as early as in the fifties. A number of qualitative laws were experimentally obtained for a plate having a bend in the generator. It was shown that the separation characteristics do not depend on the factor responsible for the phenomenon, whether it be a protrusion on the surface, a compression angle, or an incident shock. In these flows the pressure in the disturbed zone is self-induced, while the flow parameters downstream of the separation zone only affect the flow reattachment conditions and the length of the separation zone [3].

In developed separation, most of the separation zone is occupied by a low-velocity flow, in which significant pressure gradients cannot occur. Therefore, a developed separation zone is a near isobaric region with a plateau-like surface pressure distribution characterized by the value \(p_\beta\). The pressure in the separation zone is related with the viscous stress by the formula [3]

\[
(p - p_0)/p_0 - (C_f/\beta)^{1/2}, \quad \beta = (M_0^2 - 1)^{1/2}
\]

where \(p_0\) and \(C_f\) are the pressure and viscous stress on the plate at the beginning of the interaction region. Finally, the heat flux to the plate in the preseparation region immediately adjacent to the separation point (or line) is sharply reduced as compared with that for a barrier-free plate.

The most accurate solution of the separation problem was obtained within the framework of the asymptotic theory [4, 5] by the method of matched asymptotic expansions. Thus, the limiting asymptotic solution governing the flow in the vicinity of the boundary layer separation point was obtained by matching three asymptotic series for flow layers having different scale thicknesses (three-layer model). An accurate description of the mechanism of boundary layer separation from a smooth region of the body surface and the mechanism of upstream propagation of disturbances in the presence of a supersonic outer flow was made possible by taking into account the strong, though usually local, interaction of the near-wall viscous part of the flow with the outer, locally inviscid parts of the flow and the inviscid flow. Moreover, within the framework of this approach solutions were obtained not only for a small vicinity of the separation point but also for a wider class of flows characterized by a small amplitude and, at the same time, by large pressure gradients and the upstream propagation of disturbances over relatively small distances (\(\delta x \sim \text{Re}^{-3/8}\)). On the basis of this asymptotic approach, certain universal relations determining the similarity criteria for the problems under consideration were obtained.

As the Reynolds number decreases, new features appear in the gas flows. These include a strong viscosity effect throughout almost the entire disturbed flow region and surface slip and temperature jump effects, as well as the nonequilibrium nature of the flow even for translational degrees of freedom, which can influence the separation flow parameters.

In studying the aerodynamic coefficients of bodies of simple shape with a bend in the generator at low \(\text{Re} [6, 7]\) separation zones were found; however, the conditions under which such zones are formed and the features of the change in the flow parameters were not analyzed.
In this study, we consider the flow rarefaction effect on the above-mentioned and certain other features of separation flows. It is also important to establish similarity criteria and, in particular, to study the possibility of applying the similarity criteria derived for continuum flows to the case of rarefied gas flows. For the flow past non-concave bodies this question was examined, for example, in [8]. In what follows, we consider this question on the basis of the similarity criteria obtained within the framework of the asymptotic theory [9].

1. FORMULATION AND SOLUTION OF THE PROBLEM BY DIRECT STATISTICAL MODELING (MONTE-CARLO METHOD)

The problem of hypersonic flow past a flat, infinite-span plate of length \( L \) with a plane transverse barrier of height \( H \) or a compression angle at Reynolds numbers \( \text{Re}_\infty \leq 1.5 \times 10^4 \) was solved by the Monte Carlo method [10, 11]. The general flow pattern and geometry can be inferred from the density distribution (tomogram) in Fig. 1: this was obtained by solving the problem of the flow past a plate with a transverse barrier.

A detailed description of the version of the method for calculating hypersonic flows past bodies used in this study was given in [12]. Here, we will only note that in this version the real gas is replaced by an ensemble of modeling particles. The flow domain is divided into cells of the size \( d < \lambda \), where \( \lambda \) is the local mean molecular free path. The motion of the particles and the collisions between them are successively calculated for a time step \( \Delta t < \tau \), where \( \tau \) is the mean molecular free time: only particles which are contained within the same geometric cell can collide with each other. During the time interval \( \Delta t \) some of particles leave the computational domain, while a certain number of particles is introduced into the domain from the boundary in accordance with the boundary distribution function.

The problem is solved using a relaxation procedure: after a certain, fairly large number of time steps the system relaxes to a quasi-steady state, for which information on the flowfield and the forces and thermal loads acting on the body in the gas flow is gathered and analyzed. Since in the steady state the mean molecular paths may differ considerably in the different zones, it is necessary to adjust the cell dimensions so that the inequality \( d < \lambda \) is fulfilled. This adjustment is carried out by iteration: at first, a certain grid with an initial step \( d_0 \) is preassigned and a quasi-stationary solution is obtained on this grid. Then this solution is used to calculate the values of \( \lambda \) for each cell and, if required, the cell is divided in order to satisfy the above-mentioned condition \( d < \lambda \). The subdivision of the original cells proceeds in the direction of the greatest density gradient. Then the problem is solved on the new grid, and so forth, until coincidence of the solutions in successive iteration cycles is obtained. Two or three iteration cycles usually suffice for solving the problem of flow past a non-concave body.

In solving our problem, we used from 3 to 5 iteration cycles depending on the Mach and Reynolds numbers, the ratio \( h=H/L \) (or the compression angle and the lengths of its sides), and the temperature factor \( T_w/T_\infty \), where \( T_w \) is the wall temperature and \( T_\infty \) is the stagnation temperature. Depending on the governing parameters, the flowfield contained about \( 2 \times 10^5 \) cells, so that the Knudsen number based on the cell size was \( \text{Kn}_{n_e}=\lambda/d_0=2 \) at \( \text{Re}_\infty=10^4 \).

The molecular collision cross-section was determined on the basis of the model of variable-diameter spheres with a power-law particle interaction potential, \( U(r)\sim r^{-\omega} \). For this model, the viscosity-temperature dependence has the form \( \mu=\mu_0 T^{-\frac{\omega}{\gamma}} \), where \( \omega=1/2 + 2/\gamma \). Here, \( \gamma=10 \), which is a good approximation for the temperature dependence of the viscosity of nitrogen at \( T \geq 300 \text{ K} \).

We considered monatomic gas flow and the flow of a gas with rotational degrees of freedom. In the latter case, we used the Larsen-Borgnakke model [3] with a rotational relaxation parameter \( Z_R=6 \) and 3, that is, the gas may be considered to be a perfect gas with the specific heat ratio \( \gamma=1.4 \); (in what follows, both in the text and in the captions, the gas with rotational degrees of freedom is indicated, regardless of the value of \( Z_R \), by the parameter \( \gamma=1.4 \), as distinct from the monatomic gas with \( \gamma=5/3 \)). The boundary conditions on the plate surface were taken to be diffuse reflection of the molecules with an accommodation coefficient equal to unity.

In the calculation process we determined the gas parameters in the flowfield, namely, the density \( n \), the mean velocity components \( U_i \), the translational \( T_v \) and rotational \( T_R \) temperatures, and the gas temperatures in the directions of the coordinate axes

\[
kT_n = m \int (\xi - U_j)^2 f(\xi) \, d\xi
\]

where \( m \) is the molecular mass, \( \xi \) are the molecular velocity components, \( f(\xi) \) is the molecular velocity distribution function, and \( k \) is the Boltzmann constant. On the plate surface the momentum and energy fluxes