A MATHEMATICAL MODEL OF ERYTHROCYTE SEDIMENTATION IN CAPILLARIES

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A mathematical model of the process of erythrocyte sedimentation is developed that contains only two optimization parameters, namely, a rheological parameter and the hydrodynamic-resistance radius of a cell. It satisfactorily describes experimental data and can be used to judge the deformability of erythrocytes from their kinetic data and to determine the viscosity of suspensions without a viscosimeter.

Stokes's formula, which is ordinarily used to estimate the velocity of fall of a sphere in a viscous liquid ($Re \ll 1$):

$$v_0 = \frac{2gR^2 \Delta \rho}{9\eta},$$

overestimates the result for an individual erythrocyte by an order of magnitude in comparison with experiments. The velocity of motion of the interface of the medium and the erythrocyte layer (the ESR) depends strongly on the erythrocyte concentration, which is neglected by Stokes’s formula. The process of erythrocyte sedimentation is described rather thoroughly in [1]. A theoretical model is developed in Losev’s works [2]. However, it is rather complicated and we do not know experimental works based on it and other models. Losev himself indicates that with the number of optimized parameters above two the search is arbitrary. It seems desirable, first, to simplify the formalism as much as possible and, second, to include real mechanical properties of the erythrocyte in the model (the shape, volume, surface area of the membrane, deformability); then, the range of practical applicability of the model will be expanded substantially. We published the first version of a model in [3]. In the present work we describe a mathematical model of the process of erythrocyte sedimentation that is refined for the chosen geometry of a centrifuge.

Experiments were carried out using donor whole blood sampled into a hemopreservative based on sodium citrate or using suspensions of washed erythrocytes in an isotonic buffer (IB) containing 148 mM of NaCl and 5 mM of Na-phosphate buffer, pH 7.4. The blood was washed three times in an isotonic solution of NaCl at 1000 g for 600 sec. It was washed a fourth time in the IB. A paste of washed erythrocytes was diluted with the IB in various proportions for variation of the hematocrit ($H_0$) indices. Similarly, the blood was centrifuged and the paste was diluted in native plasma.

Erythrocyte sedimentation was observed in glass capillaries with a soldered lower end, $d_{in} \approx 0.004$ m. According to preliminary experiments, at $d_{in} < 0.004$ m friction of the medium against the capillary walls makes a large contribution to the ESR, and the latter depends strongly on the diameter of the capillary; on the other hand, at $d_{in} > 0.004$ m, the ESR is almost independent of the diameter of the capillary. The height of the column of blood or the suspension of erythrocytes in the capillary is $h_0 \approx 0.09$ m. Capillaries filled with blood samples were placed in an OPN-2 table centrifuge with a basket rotor and centrifuged at 17, 25, or 50 rps, stopping the centrifuge at fixed intervals of time and measuring the height $h$ of the column of settling erythrocytes. The instantaneous hematocrit was determined from the relation
Erythrocyte sedimentation in the gravitational field (1 g) was measured in vertical capillaries at room temperature of about 20°C. The viscosity of the medium was corrected for temperature nonuniformity using a formula presented in [4].

The erythrocyte concentration distribution over the height of the column in the gravitational field was evaluated, using measuring pipets with $d_{in} = 0.005$ m fastened strictly vertically. A plastic tube with a clamp was put on the lower end of the pipet. Several pipets were filled simultaneously to maximum capacity (5 ml). In intervals of 1 h the contents of a successive pipet were poured, opening the clamp, into tubes, in 10 portions of 0.5 ml each. Then, distilled water (4.5 ml) was added to the portions up to full hemolysis, and the hemoglobin was measured by an optical method using light absorption at 540 nm. The obtained optical density $D$ is proportional to the erythrocyte concentration in the sample. 4.5 ml of water was added to 0.5 ml of the initial erythrocyte suspension, and light absorption was measured to obtain $D_0$, proportional to the erythrocyte concentration in the initial suspension. The final result was the relative hemoglobin content in the samples, equal to the relative erythrocyte concentration:

$$H_b = 100 \frac{D}{D_0} \%,$$

Kinetics of erythrocyte sedimentation in the centrifuge for different hematocrits are shown in Fig. 2. At first glance they resemble exponential curves, which suggested to us and other researchers that they can be approximated by the formula

$$H = H_0 + (1 - H_0) \exp (-Kt).$$  \hspace{1cm} (3)

However, a more careful analysis of the shape of the curves showed that the initial section of the kinetics did not satisfy relation (3), although at long times $t$ the description is quite suitable (Fig. 2, dashed curves).

Erythrocyte sedimentation kinetics in native plasma and the gravitational field are somewhat different from their analogs in the IB. For the first ones, because of aggregation of erythrocytes at the very beginning of the experiment and a low ESR of individual cells in comparison with aggregates, a section of the curve with almost no slope (a lag-phase) is observed. The lag-phase is seen especially clearly at large erythrocyte concentrations, while the kinetics of erythrocyte sedimentation in the IB are rectilinear in at least the first hour of the experiment.