EFFECT OF CORIOLIS FORCES ON NONSTATIONARY INCOMPRESSIBLE VISCOUS LIQUID FLOW IN A CHANNEL

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We present a solution of the problem of nonstationary viscous incompressible liquid flow in a channel, taking account of the action of Coriolis forces. In this case the solution is shown to be determined by the sole dimensionless Taylor parameter, with an increase of which the velocity regime of nonstationary flow changes from a monotonic to a pulsating one.

The motion of a viscous incompressible liquid in noninertial coordinate systems exhibits a number of specific features, without account for which considerable errors may arise in both the description of the pattern of flow itself and calculation of its averaged hydrodynamic parameters.

Using as an example plane-parallel channel flow, we investigate the specific features of the effect of Coriolis forces on transition processes occurring in nonstationary incompressible viscous liquid flows. Problems of this kind in a stationary case were considered for the first time by Ekman to describe wind-induced oceanic flows on the rotating Earth [1, 2].

We will consider a flow of liquid moving between two infinite planes \( y = \pm h \). We will consider the constant pressure gradient to be directed along the \( OZ \) axis. Then, for a plane nonstationary stabilized flow, in which the longitudinal \( w \) and transverse (parallel to the channel walls) \( u \) velocity components are independent of the \( x \) and \( z \) coordinates, the system of dimensionless differential equations, taking account of the action of Coriolis forces, can be written in the form [3]

\[
\frac{\partial u}{\partial t} = \frac{1}{Ta} \frac{\partial^2 u}{\partial y^2} - 2w \cos \beta ,
\]

\[
\frac{\partial w}{\partial t} = - \frac{Eu}{Ta} \frac{\partial P}{\partial y} + \frac{1}{Ta} \frac{\partial^2 w}{\partial y^2} + 2u \cos \beta .
\]

The following model problem can be set to correspond with the system of equations (1). Suppose that up to a certain initial time instant the liquid was at rest. At that time instant \( t = 0 \), along the \( OZ \) axis, a constant pressure gradient appears which causes the motion of the liquid. Let us elucidate the special features of this transitional nonstationary process under conditions of the additional effect of Coriolis forces. The solution of a problem of this kind in an inertial coordinate system for the case of flow in a channel and in a round tube is given in [2, 4, 5].

Obviously, for the above problem there will correspond the following initial and boundary conditions:

\[
u \big|_{t=0} = 0; \quad w \big|_{t=0} = 0; \quad u \big|_{y=\pm 1} = 0; \quad w \big|_{y=\pm 1} = 0.
\]
Thus, we are to solve a system of inhomogeneous equations (1) under zero initial and boundary conditions (2). Of three equations of this system only the first and the third are interrelated and must be considered together. The second equation, with the values of \( u \) and \( w \) being known, is integrated easily and it serves to determine pressure.

Since the system of equations (1) is linear, the functions sought can be represented as a sum of certain particular and general solutions:

\[
\begin{align*}
    u &= u_0 + \bar{u}, \\
    w &= w_0 + \bar{w}.
\end{align*}
\]

Here the particular solution has the form

\[
\begin{align*}
    \bar{u} &= c_1 \cosh (y \sqrt{R}) \cos (y \sqrt{R}) + c_2 \sinh (y \sqrt{R}) \sin (y \sqrt{R}) - \frac{3}{2R}, \\
    \bar{w} &= c_2 \cosh (y \sqrt{R}) \cos (y \sqrt{R}) - c_1 \sinh (y \sqrt{R}) \sin (y \sqrt{R}),
\end{align*}
\]

where

\[
\begin{align*}
    c_1 &= 1.5 \frac{\cosh \sqrt{R} \cos \sqrt{R}}{R (\cosh^2 \frac{\sqrt{R}}{v} - \sin^2 \frac{\sqrt{R}}{v})}; \\
    c_2 &= 1.5 \frac{\sinh \sqrt{R} \sin \sqrt{R}}{R (\cosh^2 \frac{\sqrt{R}}{v} - \sin^2 \frac{\sqrt{R}}{v})}; \\
    R &= \frac{\omega h^2}{v} \cos \beta.
\end{align*}
\]

We will write the system of homogeneous equations to determine the functions \( u_0 \) and \( w_0 \) in the following form:

\[
\begin{align*}
    \frac{\partial u_0}{\partial t} &= \frac{\partial^2 u_0}{\partial y^2} - 2R w_0, \\
    \frac{\partial w_0}{\partial t} &= \frac{\partial^2 w_0}{\partial y^2} + 2R u_0.
\end{align*}
\]

We will seek its solution in the form

\[
\begin{align*}
    u_0 &= \exp (kt) U(y), \\
    w_0 &= \exp (kt) W(y),
\end{align*}
\]

\( k \) is a complex number. After substitution of these relations into the initial equations, we arrive at the boundary-value problem in eigenvalues for a system of ordinary differential equations:

\[
\begin{align*}
    \frac{d^2 U}{dy^2} &= kTa U + 2RW, \\
    \frac{d^2 W}{dy^2} &= kTa W - 2RU,
\end{align*}
\]

whose solution is represented in the form

\[
\begin{align*}
    U &= c_1 \exp (\lambda y), \\
    W &= c_2 \exp (\lambda y),
\end{align*}
\]

Here the constants \( \lambda, c_1, \) and \( c_2 \) are assumed to be complex numbers. After substitution of these relations into Eq. (4), we will obtain the following equation to determine \( \lambda \) and \( k \):

\[
(\lambda^2 - kTa)^2 + 4R^2 = 0,
\]

whence it follows that

\[
kTa = \lambda^2 \pm 2Ri,
\]

where \( i = \sqrt{-1} \) is an imaginary unit.

The eigenvalues \( \lambda \) are found from boundary conditions (2):