MODELLING OF THE IMPACT DYNAMICS OF LARGE METEORIC BODIES ON THE SURFACE OF THE PLANET

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We present a procedure for numerical modeling and the results predicted for dynamics of crater formation in asteroid impact on the ground in the approximation of two-dimensional gas dynamics in an axisymmetric formulation. Gas-dynamic equations are solved using a fully conservative difference scheme in Eulerian variables. Predictions are performed with both an analytic representation of the equation of state (according to Tillotson) and wide-range semiempirical equations of state with a phase transition into vapor and a more exact specification of cold compression. Consideration is given to impact at a velocity of 50 km/sec with body dimensions of the order of 1 km.

Study of the dynamics of processes occurring in a high-velocity impact is of interest for many problems of astrophysics and space physics, namely, for creating systems of meteoritic defence of space vehicles and for studying meteoritic craters, the origin of planet atmospheres, possible consequences of the fall of large space objects onto the Earth, etc. Experimental data on high-velocity impact belong to the range of velocities no higher than 20 km/sec. The range of impact velocities of the order of 100 km/sec is practically inaccessible for experimental study (the only exception being the recent fall of the Shoemakers-Levy comet onto Jupiter at a velocity over 60 km/sec). As a consequence, theoretical study of impact dynamics (especially numerical modeling) gain particular importance for the range of higher impact velocities.

Gas-dynamic equations in $r-z$ coordinates under the condition of axial symmetry of the problem are of the following form:

$$
\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial z},
$$

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z} \right) = - \frac{\partial P}{\partial r},
$$

$$\frac{1}{\rho} \frac{d\rho}{dt} = - \left[ \frac{1}{r} \frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} \right],
$$

$$\rho \frac{de}{dt} = - P \left[ \frac{1}{r} \frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} \right].
$$

The equation of state $P = P(e, \rho)$ closes the system.

As previous investigations demonstrated, modeling of dynamics of asteroid penetration into the ground and of crater formation based on numerical schemes in Lagrangian coordinates causes certain difficulties because Lagrangian cells are strongly distorted in this problem. In numerical modeling of gas-dynamic problems in variables other than Lagrangian, terms that account for convective mass, momentum, and energy transfer appear in the equations. In this case, among the most important requirements specified for numerical algorithms is adherence to

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the conservation laws and balance relations involved in differential equations. Analyzing various difference schemes used in practice reveals that difference analogs of the convective terms therein do not satisfy the conservation conditions. These schemes have additional entropy sources and sinks resulting from uncoordination of the approximation of convective flows in the equations for mass, momentum, and energy variation. Fully conservative difference schemes (FCDS) in Lagrangian variables, developed previously [1], showed good quality of the solutions for a variety of entirely different problems. Their difference from conservative ones is that some additional relations are fulfilled (a balance among individual types of energy rather than only conservation of the total energy, etc.). Attempts to work out fully conservative difference schemes for the gas-dynamic equations in Eulerian variables for a long time failed. Study [2] ascertained that such schemes cannot be constructed using the quantities determined only on two temporal layers. Only afterward was the construction of FCDS in Eulerian coordinates (three-layer over time) a success. It was shown that convective flows of mass and internal energy should be matched [3]. Subsequently, multidimensional analogs of such schemes were also obtained [4].

We use a two-dimensional partially three-layer (with respect to velocity) FCDS in Eulerian cylindrical variables (r, z) based on these principles. In the scheme, i denotes the point number along the radius r, n is the point number along the axial coordinate z, and j is the index of the temporal layer. For simplifying the description we adopted the following designation [1]: \( A = A_i^j, A = A_i^{j+1}, A = A_i^{j-1} \), and \( A = \alpha A + (1 - \alpha) A \). In the employed numerical scheme, magnitudes of the coordinates, velocities (r, z, v, w), and mass M relate to nodes of the net, and magnitudes of the density, energy, pressure, volume, and mass of the cell (\( \rho, e, P, V, m \)), to centers of the cells (these quantities are numbered by the left lower angle, i.e., \( a_{i,n} \) corresponds to \( a_{i+1/2,n+1/2} \)).

The calculation of a temporal step is broken down into two stages, as is generally accepted for Eulerian procedure. At the first stage, we solved the equation for momentum components (the tilde indicates the value of a quantity obtained after the first calculation stage)

\[
M_{i,n} \frac{[\tilde{v} + v] - (v + v)_{i,n}}{2r} = \frac{1}{2} r_i,n \left\{ (P_{i,n} - P_{i-1,n}) (z_{i,n} - z_{i,n+1}) + (P_{i,n-1} - P_{i-1,n-1}) (z_{i,n-1} - z_{i,n}) \right\},
\]

and the energy equation

\[
m_{i,n} \frac{(\tilde{e} - e)_{i,n}}{\tau} = -\frac{1}{2} P_{i,n} \left\{ ((r^{0.5})_{i+1,n} - (r^{0.5})_{i,n}) (z_{i,n+1} - z_{i,n}) + (r_{i+1,n} - r_{i,n}) (u_{i,n+1} - u_{i,n}) \right\}.
\]