Pressure Control in a Semi-Closed Volume upon Combustion of Solid Propellants with an Exponent in the Combustion Law Greater than Unity

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The problem of pressure control in a semi-closed volume by changing the critical cross-sectional area of a gas-release channel is considered upon solid-propellant combustion with the pressure, the combustion rate, and the free volume varied over a wide range (not smaller than one order of magnitude). For a system of automatic pressure control, a control algorithm is chosen and the conditions of partial parametric invariance with respect to the variable dynamic properties of the object to be controlled are formulated. The experimental results obtained upon improvement of the control system for solid rocket propellants whose exponent in the combustion law is greater than unity are given. The reasons for substantially nonstationary modes of operation of this system are considered, and a simplified model that approximates the phenomena of nonstationary combustion of a solid rocket propellant is proposed. The model is identified and the results of mathematical modeling are given. Recommendations on pressure control in the nonstationary modes of operation are given.

Solid-propellant gas generators (SPGG) are widely used as the energy source in power installations of the rocket-space industry owing to their ability to release the maximum energy per unit mass or volume for a short period of time compared to any nonnuclear energy sources. They have found application in the power-supply systems of servodrives, the feeding systems of liquid-propellant and hybrid rocket motors, the launching systems of rockets, stand gas generators for studying and testing new propellants, heat-shielding materials, and structures, the systems for controlling the thrust vector of a solid rocket motor (SRM) and reactive stabilization, and in gyroscopic devices [1-4].

The basic shortcomings of uncontrollable SPGG are the random and systematic scatters of the pressure in the chamber, the flow rate of the combustion products, and the operation period. These shortcomings are mainly removed when multimode controllable SPGG are used. Stringent requirements on the ballistic characteristics of rockets and the related problem of optimal distribution of the engine-thrust momentum in the flight time of a rocket have stimulated the development of multimode SRMs, which are controlled like SPGG. In this connection, much attention has been paid to the problems of pressure control in a semi-closed volume (combustion chamber) [1, 2, 5-8]. Technically, the problems of control of the SPGG flow rate or SRM thrust can be reduced to the problem of pressure control. Among the physical control principles realized in practice, the most frequently used principle is the variation of the critical cross-sectional area of the nozzle or channel through which the combustion products of a solid rocket propellant (SRP) are released [1, 2, 5-7].

The controllable SPGG are characterized by the use of SRPs with a strong dependence of the combustion rate on pressure, which allows one to control the flow rate over a wide range for a fixed range of pressure variation in the combustion chamber [2, 5]. The relation between the combustion rate of an SRP \( u \) and the pressure \( p \) is determined by the coefficient

\[
\nu = \frac{\partial \ln u}{\partial \ln p} = \frac{p_0 \Delta u}{u \Delta p}.
\]

In a small interval of pressure variation, the coefficient \( \nu \) is usually assumed to be constant. In this
case, the dependence \( u(p) \) is expressed by the power law
\[
\frac{\partial u}{\partial p} = \beta p^\gamma,
\]
where \( \beta \) is a constant that depends mainly on the initial temperature of the SRP charge. However, in a wide range of pressure variation, the coefficient \( \nu \) is a variable quantity [9, 10].

It is known [11] that if an SRP with \( \nu > 1 \) is used, it is impossible to ensure stable combustion in a semi-closed volume with an uncontrollable critical cross section of the gas-release channel. For a controllable SPGG, the use of these propellants narrows the stability regions and imposes additional stringent requirements on the pressure-control system. Large values of the exponent \( \nu \) (including \( \nu > 1 \)) are attained when mixed ammonium perchlorate AP-based SRPs [12] are used. The admissible maximum pressure in the SPGG combustion chamber, as a rule, does not exceed 15 MPa and is determined mainly by the requirements on the strength of the structure.

The multimode SPGG is a nonlinear nonstationary dynamic object which can be controlled only by algorithms that ensure the stability of combustion for gas-generator parameters (especially, the pressure in the combustion chamber and its free volume) varying in a wide range (not smaller than one order). The control system for multimode SPGG is usually designed with a view to reaching operating conditions under which the dynamics of pressure variation agrees well with the quasistationary mathematical model that describes the variation in the volume-average pressure and gas temperature on the basis of the law of conservation of mass and energy of the SRP combustion products in a semi-closed volume. The quasistationary model from [13] describes satisfactorily the processes that occur in the SPGG combustion chamber with a rate of pressure variation of up to 2 MPa/sec. This rate is reached by choosing the control algorithms and the form of control signals.

In the SPGG pressure-control system, the following proportional-integral control law is widely used [6]:
\[
U_c = a_0 \Delta p + a_1 \int_{0}^{\tau_c} \Delta p \, dt,
\]
where \( \Delta p = p - p_{sp} \) is the pressure-control error, \( p_{sp} \) and \( p \) are the specified and actual pressures in the SPGG combustion chamber, respectively; \( a_0 \) and \( a_1 \) are the coefficients of the control law, and \( U_c \) is the control signal at the drive of the system drive at the moment \( \tau_c \).

The coefficients \( a_0 \) and \( a_1 \) can be chosen by the frequency methods [14] for constant values of the SPGG parameters, which correspond to the stationary mode of operation in the steady gas formation and outflow. The transfer function of an SPGG [1, 2, 15]
\[
W_{sp}(s) = - \frac{k_{sp}}{\tau_{sp} s + 1}
\]
corresponds to this case, where
\[
k_{gg} = \varphi B(k)p_0/S \rho \sqrt{R_T} \left[ \frac{u_0}{p_0} - \left( \frac{\partial u}{\partial p} \right)_0 \right],
\]
\[
\tau_{sp} = V_0/S \rho \tau T_T \left[ \frac{u_0}{p_0} - \left( \frac{\partial u}{\partial p} \right)_0 \right];
\]
\[
A = \varphi B(k)/\sqrt{R_T} T_T;
\]
\[
B(k) = \sqrt{2k/(k - 1)} \left( \frac{\beta_k}{\beta_k^{(k+1)/k}} \right);
\]
\[
\beta_k = \left( \frac{2}{k + 1} \right)^{k/(k-1)};
\]
\( \rho, R, k \) are the density, universal gas constant, and adiabatic exponent of the SRP combustion products, \( \varphi \) is the flow-rate coefficient, \( p_0 \) and \( V_0 \) are the stationary (equilibrium) values of the pressure and the free volume, respectively, \( S \) is the area of the SRP combustion surface, \( T_p \) is the isobaric equilibrium temperature, \( \chi \) is the heat-loss coefficient, \( u_0 \) and \( \left( \frac{\partial u}{\partial p} \right)_0 \) are, respectively, the combustion rate of the SRP and its partial derivative with respect to the pressure \( p_0 \), and \( s \) is the Laplace operator.

For the control law (2), the transfer function of the control device has the form
\[
W_c(s) = a_1 \left( \frac{a_0}{a_1} \right) \frac{s + 1}{s}.
\]

Analyzing the product of the transfer functions (3) and (5), we infer that the dynamic properties of the pressure-control system are stable upon variation of the SPGG parameters if the following conditions hold:
\[
\frac{a_0}{a_1} = \tau_{gg} = V_0/S \rho \tau T_T \left[ \frac{u_0}{p_0} - \left( \frac{\partial u}{\partial p} \right)_0 \right],
\]
\[
k_0 = k_0 k_r k_0 a_1
\]
\[
= k_d k_r k_0 a_1 p_0/S \rho \left[ \frac{u_0}{p_0} - \left( \frac{\partial u}{\partial p} \right)_0 \right], = \text{const.}
\]
Here \( k_d, k_r, \) and \( k_0 \) are the transfer coefficients of the drive, the SPGG control element, and the pressure