A global excitation variable is introduced, as a conformal factor of the metric that depends only on time, into the Einstein theory of gravitation formulated in terms of conformally invariant variables. The dynamics of this global excitation is isolated from the Einstein equations by direct averaging of their dynamical part over large spatial volumes. The conditions are found under which this dynamics duplicates the dynamics of the Friedmann cosmological model.

1. Introduction

A cosmological model has recently been proposed [1-3], the essence of which consists in the choice of conformal variables and the complete separation of the dynamical degrees of freedom from the nonphysical variables associated with the reparameterization of time. This separation is achieved by a canonical transformation of the scale factor and of its associated momentum [4, 5]. As a result, coupling of the first kind becomes a linear function of the new momentum. Exact resolution of such coupling enables one to identify the new momentum with the energy of a material source, while the new scale factor is an invariant time parameter of evolution.

Such a conversion of a component of the metric tensor into "time" solves the problem of normalizability of the wave function and enables one to establish a connection with the data of observational cosmology [2, 3].

In the present work, using the Arnowitt-Deser-Misner (ADM) parameterization [6, 7] and Lichnerowicz conformally invariant field variables [8-10], a conformal factor of the metric that depends only on time is isolated from the metric of the general theory of relativity (GTR) and its dynamics is analyzed.

2. Model and Variables

We consider a system of gravitational and electromagnetic fields that emerges from the Hilbert-Einstein action

$$W = \int d^4x \sqrt{-g} \left[ -\mu^2 \frac{(4)R(g)}{6} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] = \left( \mu^2 = M_{pl} \frac{3}{8\pi} \right)$$

(1)
(\(F_{\mu\nu}\) is the electromagnetic field tensor and \(M_{Pl}\) is the Planck mass). This action is invariant with respect to general coordinate transformations:

\[ x_\mu \Rightarrow x'_\mu(x_0, x_1, x_2, x_3). \]  

(2)

In the Hamiltonian approach, it is convenient to choose the well-known parameterization of the metric [6, 7]

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu = N^2dt^2 - g^{(3)}_{ij}dx^i dx^j; \quad \tilde{dx}^i = dx^i + N^i dt, \]

(3)

where \(N\) is the displacement function and \(N^i\) is the displacement vector. Such parameterization reduces the explicit invariance of the action (1) with respect to (2) to invariance with respect to the kinemetric transformations [7]

\[ t \Rightarrow t' = t'(t); \quad x_i \Rightarrow x'_i = x_i(t, x_1, x_2, x_3). \]

(4)

We introduce into (1) the conformally invariant variables [8-10]

\[ h_{ij} = \left( \frac{g^{(3)}_{ij}}{g^{(3)}} \right)^{\frac{1}{3}} g^{(3)}_{ij}; \quad \sqrt{h} = 1 \]

(5)

and in accordance with this we rewrite a space-time interval in the form

\[ (ds)^2 = a^2(t, x)\left[N^2dt^2 - h_{ij}dx^i dx^j\right]; \quad a = g^{(3)} \frac{1}{\sqrt{h}}, \]

(6)

where the square of the spatial scale \(a(t, x)\) is also a conformal factor.

The action (1) is written in terms of the variables \([a, N, N^k, h_{ij}]\) as follows:

\[ W = \int_{t_i}^{t_f} dt \int d^3x \left[ L_w + L_h + L_A + L_{(S)} \right]. \]

(7)

where the spatial surface term is

\[ L_{(S)} = -\mu^2 \left[ \partial_k \left(N^k a a^2 \right) + \frac{1}{3} \partial_k \left(a \partial^k (aN_c) \right) \right], \]

(8)

while

\[ L_w = \mu^2 \left[ \partial_0 \left( a^2 \right) - N_c a^2 \right] \]

\[ a = \frac{1}{N_c} \left( \dot{a} - N^k \partial_k a - a \partial_k N^k \right), \]

(9)

\[ L_h = \frac{\mu^2}{6} a^2 N_c \left( \frac{\dot{h}}{4} - \tilde{R}(h) \right) \]

\[ \tilde{R} = (3) R(h) + 8 a^{-1/2} \Delta a^{1/2}, \]

(10)

\[ L_A = N_c \frac{1}{2} \left[ A^2 - \frac{1}{2} F_{ij} F^{ij} \right] \]

\[ \dot{A}_k = \frac{1}{N_c} \left( \dot{A}_k - \partial_k A_0 - N^i F_{ik} \right), \]

(11)

Into these expressions we have introduced the kinemetrically invariant time derivative

\[ h_{ij} = \frac{1}{N_c} \left( \dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i + \frac{2}{3} h_{ij} \partial_k N^k \right), \]

\[ h^2 = h_{ij} h^{ij}, \quad \dot{A}^2 = \dot{A}_k A_l h^{kl}, \]

(12)

\(\nabla_i\) is a covariant derivative in the metric \(h_{\mu\nu}\), and \(\Delta (...) = \nabla_i \partial^i (...).\)