SOLUTION OF THE PROBLEM OF THE STRESS STATE OF NONCIRCULAR CYLINDRICAL SHELLS OF VARIABLE THICKNESS

Ya. M. Grigorenko and L. I. Zakhariuchenko

A procedure is proposed for solving two-dimensional boundary-value problems on the stress-strain state of open and closed noncircular cylindrical shells of variable thickness under surface loads. The solution is based on the use of the spline-collocation method along the directrix and the method of discrete orthogonalization along the generatrix. Examples of solutions for ellipsoidal shells of variable thickness are given.

The problem of the stress-strain state of noncircular cylindrical shells of variable thickness is described by a system of partial differential equations with variable coefficients and corresponding boundary conditions [3, 13]. This class of problems can be examined in a simplified formulation on the basis of the Mushtari-Donnel-Vlasov equations [1, 4, 12]. In many cases, the use of various approximate and numerical methods to solve such problems does not make it possible to satisfy the boundary conditions with sufficient accuracy and obtain the desired solution [2, 6, 9].

In this investigation, we solve problems of the given type on the basis of spline approximation in one coordinate direction and the numerical method of discrete orthogonalization in the other coordinate direction.

This approach to solving boundary-value problems of shell theory was proposed in [5, 7, 8]. Some results that have been obtained from solving problems for circular and noncircular cylindrical shells of constant thickness can be found in [5, 11].

Here, we examine the class of problems concerning the stress-strain state of noncircular isotropic thin cylindrical shells with a thickness that changes along the generatrix and directrix. The shells are subjected to an arbitrary surface load. The solution is obtained on the basis of the Mushtari-Donnel-Vlasov equations [1, 12]. The shells may be closed or open along the generatrix. Accordingly, the boundary conditions are assigned either on the curvilinear edges or over the entire contour.

The initial equations that describe the deformation of this class of shells in the coordinate system s, t — where s and t are arc length along the generatrix and directrix, respectively — are written in the following form [1, 4, 12]:

the expressions for the strains

\[ \varepsilon_s = \frac{\partial u}{\partial s}, \quad \varepsilon_t = \frac{\partial u}{\partial t} + \frac{w}{R}, \quad \varepsilon_{st} = \frac{\partial u}{\partial t} + \frac{\partial v}{\partial s}, \]

\[ \kappa_s = -\frac{\partial^2 w}{\partial s^2}, \quad \kappa_t = -\frac{\partial^2 w}{\partial t^2}, \quad \kappa_{st} = -\frac{\partial^2 w}{\partial s \partial t}, \]

the equations of equilibrium

\[ \frac{\partial N_s}{\partial s} + \frac{\partial S}{\partial t} = 0, \quad \frac{\partial N_s}{\partial s} + \frac{\partial N_t}{\partial t} = 0. \]
\[
\frac{\partial Q_s}{\partial s} + \frac{\partial Q_t}{\partial t} - \frac{1}{R} N_t + q_t = 0, \tag{2}
\]
\[
\frac{\partial M_s}{\partial s} + \frac{\partial H}{\partial t} - Q_s = 0, \quad \frac{\partial M_t}{\partial t} + \frac{\partial H}{\partial s} - Q_t = 0,
\]
the elasticity relations

\[
N_s = D_N (e_s + v e_t), \quad N_t = D_N (e_t + v e_s), \quad S = \frac{1 - v}{2} D_N \varepsilon_{st},
\]

\[
M_s = D_M (\kappa_s + v \kappa_t), \quad M_t = D_M (\kappa_t + v \kappa_s), \quad H = (1 - v) D_M \kappa_{st}, \tag{3}
\]
where the radius of curvature of the directrix \( R = R (t) \), while the stiffnesses

\[
D_N = \frac{E h (s, t)}{1 - v^2}, \quad D_M = \frac{E h^3 (s, t)}{12 (1 - v^2)}.
\]

In Eqs. (1)–(3), \( u, v, \) and \( w \) are the displacements along the generatrix, the directrix, and a normal to the middle surface; \( e_s, e_t, e_{st}, \kappa_s, \kappa_t, \) and \( \kappa_{st} \) are the shear and bending strains; \( N_s, N_t, S, Q_s, \) and \( Q_t \) are forces; \( M_s, M_t, \) and \( H \) are moments; \( h = h (s, t) \) is the thickness of the shell; \( E \) and \( v \) are the elastic modulus and Poisson's ratio; \( q_t = q_t (s, t) \) is the surface load.

After certain transformations, Eqs. (1)–(3) lead to a resolvent system of equations in displacements

\[
D_N \left[ \frac{\partial^2 u}{\partial s^2} + \frac{\partial u}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial u}{\partial t} + \frac{\partial v}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} \right] = 0,
\]

\[
D_N \left[ \frac{\partial^2 v}{\partial s^2} + \frac{\partial v}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial u}{\partial t} + \frac{\partial w}{\partial s} \right] = 0,
\]

\[
D_N \left[ \frac{\partial^2 w}{\partial s^2} + \frac{\partial w}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial w}{\partial t} + \frac{\partial v}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial u}{\partial t} + \frac{\partial v}{\partial s} \right] = 0,
\]

\[
D_M \left[ \frac{\partial^2 w}{\partial s^2} + \frac{\partial w}{\partial t} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial w}{\partial t} + \frac{\partial v}{\partial s} \right] + \frac{1 - v}{2} \frac{\partial}{\partial t} \left[ \frac{\partial u}{\partial t} + \frac{\partial v}{\partial s} \right] = q_t, \quad 0 \leq s \leq L, 0 \leq t \leq 2 \pi. \tag{4}
\]

In the case of shells that are closed along the directrix, the boundary conditions are assigned on the curvilinear edges. In the case of shells that are open along the directrix, the boundary conditions are assigned on the curvilinear and straight edges. The boundary conditions can be formulated in displacements or in mixed form. Arbitrary boundary conditions are assigned on the curvilinear edges, while the following boundary conditions are assigned on the straight edges in the case of open cylindrical shells: conditions corresponding to pinned or fixed support on both edges, specifically:

\[
u = v = w = 0 \quad \text{at} \quad t = t_1, \quad t = t_2 \tag{5}
\]
or

\[
u = v = w = M_t = 0 \quad \text{at} \quad t = t_1, \quad t = t_2; \tag{6}
\]

conditions corresponding to pinned support of one end (6) and fixed support of the other end (5). Generally speaking, other types of boundary conditions can also be assigned on the straight edges.

Using certain transformations, we write resolvent system of differential equations (4) in the form: