DATA RECONCILIATION FOR INPUT-OUTPUT MODELS IN LINEAR DYNAMIC SYSTEMS

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Abstract—Sequential data reconciliation algorithms have been developed for input-output models in linear dynamic systems. Existing filtering methods do not treat the case where there are measurement errors in the input variables. In our approach, the measurement errors in the input variables are optimally handled by the least squares method. This method shows good performance for input-output models.

Key words: Data Reconciliation, Least-squares Estimation

INTRODUCTION

Data reconciliation has received considerable attention to resolve inconsistencies between plant measurements and balance equations. A number of commercial data reconciliation software packages are available for steady-state processes [Ayral, 1994]. In order to enhance robustness of the calculations, the data reconciliation step can be accompanied by gross error detection [Kim et al., 1995].

Most previous research on data reconciliation has focused on linear steady-state systems. During the last several years, there have been several approaches proposed for nonlinear dynamic systems. Data reconciliation for nonlinear dynamic systems was treated by Darouach and Zasadzinski [1991], Liebman et al. [1992] and Ramamurthi and Bequette [1993]. Darouach and Zasadzinski [1991], and Rollins and Devanathan [1993] suggested data reconciliation algorithms for linear dynamic systems. A nonlinear data reconciliation program has been applied to reconcile actual plant data from a feed-blend tank of an Exxon Chemicals plant [McBrayer, 1994].

In this paper we employ a model used in Darouach and Zasadzinski, which has different discrete equations of the form EXk+1 = BXk. This is called a singular or generalized dynamic model, because the matrix E is singular and therefore the standard Kalman filter cannot be applied.

In this article, sequential data reconciliation algorithms are developed for input-output models in linear dynamic systems. Unlike other filtering methods, the measurement errors in the input variables are optimally handled by least squares.

PROBLEM STATEMENTS

1. Linear Difference Equation

Consider a dynamical system with input signal \{u(t)\} and output signal \{y(t)\}. Suppose that these signals are sampled in discrete time \(t=1,2,3,\ldots\) and that the sampled values can be related through a linear difference equation. The general nth order difference equation relating the input \(u(k)\) and output \(y(k)\) is

\[
y(k) + a_1y(k-1) + \cdots + a_ny(k-n) = b_1u(k) + b_2u(k-1) + \cdots + b_nu(k-n)
\]

or

\[
y(k) + \sum_{j=1}^{n} a_jy(k-j) = \sum_{j=0}^{n} b_ju(k-j)
\]

where \(k\) is the integer time index, and \(a_j\) and \(b_j\) are the constant coefficients.

The representation of Eq. (1) can be extended to systems that have multiple inputs and multiple outputs. Suppose there are \(m\) inputs and \(r\) outputs, and that the vectors \(U(k)\) and \(Y(k)\) are defined as

\[
U(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix}, \quad Y(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_r(k) \end{bmatrix}
\]

The system can then be represented by the vector difference equation

\[
Y(k) + \sum_{j=1}^{n} A_jY(k-j) = \sum_{j=0}^{n} B_jU(k-j)
\]

in which \(A_j\) and \(B_j\) are constant coefficient matrices of dimension \(r \times r\) and \(r \times m\) respectively.

2. Data Reconciliation in Linear Dynamic Systems

Here we consider the problem of estimating the vector \(X\), at time instants \(i=1,2,\ldots,k+1\). From Eqs. (1) and (2) we can collect the \((k+1)\) measurements and the \(k\) constraints as follows [Darouach and Zasadzinski, 1991]:

\[
Z = X^* + \varepsilon
\]

\[
\Phi X^* = 0
\]

where \(X^*\) is a vector of true values of the variables and \(\varepsilon\) is a vector of normally distributed random measurement noise with zero mean and known covariance matrix.

Using this notation, the dynamic data reconciliation problem can be formulated analogous to the steady-state case, that is.
Min \[ \frac{1}{2} (\hat{x} - Z)^{T} V^{-1} (\hat{x} - Z) \]  
subject to  
\[ \Phi \hat{x} = 0 \]

where \( V \) is the variance-covariance matrix of measurements. The solution of this problem is given in many references [e.g., Darouach and Zasadzinski, 1991; Kuehn and Davidson, 1961]. For dynamic data reconciliation problems, a recursive solution based on the sequential method developed for steady-state case [Darouach et al., 1988] can be applied.

**DERIVATION OF ESTIMATION ALGORITHMS**

The dynamic data reconciliation problem can be solved using the steady-state sequential method [Darouach and Zasadzinski, 1991]. Their algorithms are applied to input-output models for linear dynamic systems.

1. **First Order Model**

\[ y(k+1) + a_1 y(k) = b_1 u(k+1) + b_2 u(k) \]  

The above equation can be written

\[ EX_{k+1} = BX_k \]  

where

\[ X_k = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \]  
\[ E = \begin{bmatrix} 1 & -b_2 \\ 0 & 1 \end{bmatrix} \]  
\[ B = \begin{bmatrix} -a_1 & b_1 \\ 0 & a_1 \end{bmatrix} \]

The reconciled estimates are calculated by the equations below.

\[ \hat{x}_{k+1} = \hat{x}_k + \Delta t \hat{y}_k \]  
\[ \hat{y}_{k+1} = (1 - b_2 \Delta t) \hat{y}_k + b_1 \Delta t \hat{u}_k \]  
\[ \hat{u}_{k+1} = \frac{1}{a_1 \Delta t + b_2 \Delta t + \Delta t} \]

\[ \Sigma_k = V - VE^T G E \]  

2. **Second Order Model**

\[ y(k+1) + a_1 y(k) + a_2 y(k-1) = b_1 u(k+1) + b_2 u(k) + b_3 u(k-1) \]

The above equation can be written

\[ EX_{k+1} = BX_k + AX_k \]  

where

\[ X_k = \begin{bmatrix} y(k) \\ u(k) \\ y(k-1) \end{bmatrix} \]  
\[ E = \begin{bmatrix} 1 & 0 & 0 \\ -b_3 & 1 & 0 \\ 0 & -b_2 & 1 \end{bmatrix} \]  
\[ A = \begin{bmatrix} -a_2 & b_1 & b_2 \\ -a_1 & b_1 & b_3 \\ 0 & a_1 & b_2 \end{bmatrix} \]

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\[ \hat{y}_{k+1} = (1 - b_2 \Delta t) \hat{y}_k + b_1 \Delta t \hat{u}_k \]  
\[ \hat{u}_{k+1} = \frac{1}{a_1 \Delta t + b_2 \Delta t + b_3 \Delta t + \Delta t} \]

\[ \Sigma_k = V - VE^T G E \]

**SIMULATION RESULTS**

1. **First Order Model**

\[ G(s) = \frac{1}{s+1} \]

The first order model can be discretized exactly for a piecewise constant input [Seborg et al., 1989].

\[ y(k+1) + a_1 y(k) = b_1 u(k) \]

The estimates can be calculated sequentially with the initial conditions \( X_0 = y_0 \) and \( Z_0 \).

\[ \hat{y}_{k+1} = (1 - b_2 \Delta t) \hat{y}_k + b_1 \Delta t \hat{u}_k \]  
\[ \hat{u}_{k+1} = \frac{1}{a_1 \Delta t + b_2 \Delta t + \Delta t} \]

where

\[ \Sigma_k = V - VE^T G E \]

2. **Second Order Model**

\[ y(k+1) + a_1 y(k) + a_2 y(k-1) = b_1 u(k+1) + b_2 u(k) + b_3 u(k-1) \]

The above equation can be written

\[ EX_{k+1} = BX_k + AX_k \]  

where

\[ X_k = \begin{bmatrix} y(k) \\ u(k) \\ y(k-1) \end{bmatrix} \]  
\[ E = \begin{bmatrix} 1 & 0 & 0 \\ -b_3 & 1 & 0 \\ 0 & -b_2 & 1 \end{bmatrix} \]  
\[ A = \begin{bmatrix} -a_2 & b_1 & b_2 \\ -a_1 & b_1 & b_3 \\ 0 & a_1 & b_2 \end{bmatrix} \]

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\[ \hat{x}_{k+1} = \hat{x}_k + \Delta t \hat{y}_k \]  
\[ \hat{y}_{k+1} = (1 - b_2 \Delta t) \hat{y}_k + b_1 \Delta t \hat{u}_k \]  
\[ \hat{u}_{k+1} = \frac{1}{a_1 \Delta t + b_2 \Delta t + b_3 \Delta t + \Delta t} \]

The data were simulated by a random number generator. By the use of the method proposed here the input and output variables can be filtered simultaneously as shown in Fig. 1 (76% error reduction in y) and Fig. 2 (64% error reduction in u). Tables 1 and 2 shows the error reductions of input and output variables as a function of the ratio of standard deviations. As \( \sigma_u/\sigma_y \) is decreased from 20 to 0.05, the error reduction in y decreases exponentially from 92.3% to 1.7%, and the error reduction in u is increased exponentially from 0.3% to 73%. As shown in Fig. 3, the measurement errors of the output variable are reduced exponentially as \( \sigma_u/\sigma_y \) is increased. In other filtering problems, the input variables are assumed to be error-free. But in practical situations, measurements may include random and/or gross errors. In the worst case, there could be small errors in output variables, but large errors in input measurements. If large errors in the input variables are ignored, the filtered estimates from other methods lose the reliability. Fig. 4 shows the reconciled estimates of the input variable when the input is assumed to be error-free, even though there exists a relatively large error in the input var-