MODEL PREDICTIVE CONTROL FOR MULTIVARIABLE UNSTABLE PROCESSES WITH CONSTRAINTS ON MANIPULATED VARIABLES

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Abstract—The original MPC (Model Predictive Control) algorithm cannot be applied to open loop unstable systems, because the step responses of the open loop unstable system never reach steady states. So when we apply MPC to the open loop unstable systems, first we have to stabilize them by state feedback or output feedback. Then the stabilized systems can be controlled by MPC. But problems such as valve saturation may occur because the manipulated input is the summation of the state feedback output and the MPC output. Therefore, we propose Quadratic Dynamic Matrix Control (QDMC) combined with state feedback as a new method to handle the constraints on manipulated variables for multivariable unstable processes. We applied this control method to a single-input-single-output unstable nonlinear system and a multi-input-multi-output unstable system. The results show that this method is robust and can handle the input constraints explicitly and also its control performance is better than that of others such as well tuned PI control, Linear Quadratic Regulator (LQR) with integral action.

INTRODUCTION

The universal drive for more efficient use of energy in the chemical and allied industries has resulted in the imposition of stricter demands on control systems. For effective control, the control system must cope with the problems caused by time delays, interactions among system variables, and inherent system nonlinearities. In addition, it must be capable of handling constraints in the input variables as well as in the output variables while remaining robust in the face of modeling errors, measurement noise, and unmodeled disturbances.

The steps taken by the Shell Oil Company (U.S.A.) towards solving the above problems led to the development of the Dynamic Matrix Control (DMC) technique which first appeared in the open literature in Cutler and Ramaker (1979) after having been applied with notable success on industrial processes since 1973.

In 1986, an extended method for the solution of the DMC problem was introduced. The method denoted as QDMC (Quadratic Dynamic Matrix Control) consists of the on-line solution of a quadratic programming (QP) problem which minimizes the sum of squared deviations of controlled variable projections from their setpoints subject to maintaining projections of constrained variables within bounds (Morshed, et al., 1986).

For open loop unstable systems, the discrete representation of the dynamics is impossible because open loop unstable systems can never reach steady state. Therefore, the original DMC algorithm cannot handle the open loop unstable systems. In 1990, model predictive control combined with coordinate control strategies was proposed to accomplish the control of a nonlinear, open-loop unstable process (P.M. Hidalgo and C.B. Brosilow [9]). This algorithm computes the manipulated variables so that the model output exactly tracks the desired model output at the next time horizon by solving a Newton’s algorithm. But this method cannot prevent the saturation of manipulated variables explicitly and would be unstable in the case of severe plant/model mismatch though large filter time constant of the reference trajectory equation can be used to stabilize the system.

In this study, we propose QDMC combined with state feedback as a new method to handle constraints of controlled variables and manipulated variables for open loop unstable systems. This method has a similar structure to original QDMC except for constraint equations that include state feedback information. We compare the performance of the proposed method with
that of Linear Quadratic Regulator (LQR) with integral action.

**THEORETICAL BACKGROUND**

1. QDMC Combined with State Feedback

The philosophy of Quadratic Dynamic Matrix Control (QDMC) is well described by Garcia and Morshedli [7]. Here, we express a simple quadratic solution of DMC and next derive the QDMC constraint equation combined with state feedback. One can express the least-squares solution of the DMC equations as the following quadratic minimization problem:

\[
\min_{u(k)} \frac{1}{2} \left[(Au(k) - e(k + 1))^T \Gamma (Au(k) - e(k + 1)) + \frac{1}{2} u(k)^T \Lambda' \Lambda u(k)\right]
\]

\[
\text{s.t. } Cu(k) \geq g(k + 1),
\]

where \(A\) : dynamic matrix

\(u(k)\) : manipulated input vector at present time,

\(\Delta e(k) = \Delta l(k + 1), l = \text{control horizon}\)

\(e(k + 1)\) : predicted error vector

\(\Gamma\) : weighting matrix of controlled variables

\(\Lambda\) : weighting matrix of manipulated variables

Here, dynamic matrix \(A\) is comprised of the step response coefficients for the inner closed loop system. The above quadratic problem can be simplified as following:

\[
\min_{u(k)} \frac{1}{2} u(k)^T Hu(k) - g(k + 1)^T u(k)
\]

\[
\text{s.t. } Cu(k) \geq g(k + 1),
\]

where

\[
H = A^T \Gamma A - \Lambda' \Lambda \quad \text{(the QP Hessian matrix)}
\]

\[
g(k + 1) = A^T \Gamma Tr(k + 1) \quad \text{(the QP gradient vector)}
\]

Next thing we have to do is to make the linear inequality constraints matrix. In the following we show how the inequalities are formulated for each of the constrained variables for a s-input r-output system.

1-1. Manipulated Variables

In QDMC, the vector \(u(k)\) contains not only the present moves to be implemented but also predictions of the future moves. But if we want to bound the range of manipulated variables, we do not have to limit all the elements of \(u(k)\), because only \(\Delta l(k)\) is implemented and \(u(k)\) is recalculated at the next time. In the case of QDMC combined with state feedback, final control action is the summation of \(l(k)\) and state feedback, that is \(l(k) - Kx(k)\) where \(K\) is the \(i\)th row vector of the inner loop state feedback gain and \(x(k)\) is the vector of the present state value. LQR can be used to calculate the state feedback gain matrix \(K\). So one can bound the predicted level of the \(i\)th manipulated variable as follows:

\[
l_{min} \leq l(k) + \Delta l(k) - Kx(k) \leq l_{max}
\]

\[
l_{min} \leq l(k) + \Delta l(k) - Kx(k) \leq l_{max}
\]

where \(l(k)\) is the present value of the \(i\)th manipulated variable; and \(l_{min}, l_{max}\) are the lower and upper limits respectively. In matrix form, these constraints are expressed as:

\[
\begin{bmatrix}
-1 & \cdots & -1 \\
1 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
l(k) \\
\end{bmatrix}
\geq
\begin{bmatrix}
[I_l(k) - I_{l_{max}}] - Kx(k) \\
[I^T l(k) - I_{l_{max}}] - Kx(k) \\
[I^2 l(k) - I_{l_{max}}] + Kx(k) \\
[I^3 l(k) - I_{l_{max}}] + Kx(k)
\end{bmatrix}
\]

1-2. Controlled Variables

Dynamic Matrix is made of the step response coefficients of the inner closed loop system. So the constraint equations of controlled variables are the same as before. For a single output system, with respective maximum and minimum limits \(O_{max}, O_{min}\), the constraint equations are formulated as:

\[
\begin{bmatrix}
-A \\
A
\end{bmatrix}
\begin{bmatrix}
u(k) \\
\end{bmatrix}
\geq
\begin{bmatrix}
(O - O_{max}) \frac{1}{2} e(k + 1) \\
(O_{max} - O) \frac{1}{2} e(k + 1)
\end{bmatrix}
\]

where \(1 = [1 \ 1 \ \cdots \ 1]\)

Extension to the multiple-output case is straightforward.

\[
\begin{bmatrix}
-A \\
A
\end{bmatrix}
\begin{bmatrix}
u_l(k) \\
\end{bmatrix}
\geq
\begin{bmatrix}
(O_{l_{max}} - O_{l_{min}}) \frac{1}{2} e_l(k + 1) \\
(O_{l_{max}} - O_{l_{min}}) \frac{1}{2} e_l(k + 1)
\end{bmatrix}
\]

where \(O_{l}\) is the value of the \(r\)-th controlled variable setpoint; \(O_{l_{min}}\) and \(O_{l_{max}}\) are the maximum and the minimum limit of the \(r\)-th controlled variable respectively.

The simplified block diagram of QDMC combined with state feedback is shown in Fig. 1. In the figure, we include a state estimator to predict the states of the system and simply represent it as a state estimator block to avoid complexity. If Kalman filter is used as the state estimator, then the stability and robustness of the system would be improved. But here, the state estimator is not important. So we do not mention