A Two-Sector Model of Production and Trade with Variable Factor Supplies

By

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I. Introduction

The algebraic, two-sector, general equilibrium model [e.g., Harberger, 1962; Jones, 1965] has gained wide acceptance for describing the behavior of the neo-classical model of production. The implications and proofs of this model for the Stolper-Samuelson Theorem, the Rybczynski Theorem, factor price equalization etc., are by now familiar to most who have studied pure trade theory. However, an underlying assumption of this model is that the supplies of capital and labor available to the market are inelastic; that is, the total supply of these factors do not respond to changing factor rewards. Yet, when the wage/rental ratio of the model changes, it is possible that this relative factor reward change is comprised of an absolute change in the reward to one or both factors. Where absolute rewards also change, the model should produce not only a movement of factors from one industry to the other, but also an overall expansion or contraction of the total quantity of factors available to the market. This expansion effect in turn can reinforce or offset the usual relative output effects.

The effects of changes in absolute factor rewards have recently received increased attention in the economic literature in terms of the “wedge” model of national production. Attention in that model has been focused in particular on the effects of tax rates or other wedges on factor supplies, and thus the level of market output within individual countries.

In this paper we examine how the variable factor supply model changes the results and implications of the traditional 2 × 2 model of the open economy. In Section II we introduce a household production function as a means of generating our variable labor supply model. In

1 See for example, Boskin [1978], Jones [1979], Evans [1978], Canto et al. [1978], and Canto and Miles [1981].
the next section we examine the new relationship between market production and changes in the wage/rental ratio, which accounts for corresponding changes in labor supply to the market sector. We explain how under some very general conditions, the relationship between the two market-produced goods is no longer concave to the origin. This seemingly perverse result has a very rational explanation, and in the final section it provides the basis for a plausible, alternative explanation of the Leontief Paradox.

II. The Basic Model

As in the traditional model, two factors of production, capital (K) and labor (L), are assumed to be combined in the production process to produce two market goods X₁ and X₂. In addition, following Becker [1965], it is assumed that a household commodity Z is produced by combining market-purchased good X₂ and household (leisure) time L. Individuals in the household must therefore decide how to allocate their time between market activities (L₁ and L₂) for which they are paid, and household activities (L) which produce commodities (Z) that they can consume directly.

Since this household commodity Z is produced and consumed in the household, it is truly a nontraded good. For simplicity it is assumed that Z is the only source of utility, and that the only use of X₂ is in the production of Z. X₂ is therefore relegated to the role of an intermediate good in utility production. The remaining good X₁ can be considered an investment good used to produce X₁ and X₂ in future periods.

The market and nonmarket production functions are assumed to be linear homogeneous, continuous, twice differentiable and to require both inputs, that is

\[ X_i = F_i (K_i, L_i) = L_i \cdot f_i (\theta_i), \quad i = \{1, 2\} \]

\[ Z = F_Z (X_2, L) = L \cdot f_Z (\psi) \]

where \( \theta_i = K_i/L_i \) denotes the capital/labor ratio in the production of market goods and \( \psi = X_2/L \) denotes the ratio of market goods to nonmarket time used in the production of the household commodity.

The household commodity capital intensity (\( \theta_Z \)) can be expressed in terms of the capital intensity of consumption good X₂ (\( \theta_2 \)):¹

¹ Differentiating logarithmically equation (3)

\[ E\theta_Z = E\psi + E\theta_2 \]

\[ = EL_2 \{1 - [L/(L_2 + L)]} - [L/(L_2 + L)] EL + E\theta_2 \]

\[ = [L/(L_2 + L)] (EL_2 - EL) + E\theta_2 \]

continued