
By R. H. Fowler and E. A. Milne.

(Communicated by the Director of the Solar Physics Observatory.)

§ 1. In a previous paper† we have applied the theory of high-temperature ionisation, first developed by Saha, to the positions of maximum intensity of absorption lines in the stellar sequence of spectra. During the past year the series relationship of lines belonging to trebly-ionised silicon and to ionised carbon, which are important in the spectra of the hotter stars, have been discovered by A. Fowler. ‡ It is the object of the present paper to use A. Fowler’s results to calculate by the method of maxima the temperatures and pressures in the reversing layers of the hotter stars. We can supplement these results by provisional estimates for certain other spectra. We deal also with a number of other points which arise in applications of the theory.

§ 2. Résumé of the Method.—We use the terms “principal series” or “principal lines” to denote series or lines which are absorbed in transitions from the normal state of a neutral, singly-ionised, or multiply-ionised atom to an excited state, or state of higher energy. We use the

* Monthly Notices, 84, No. 5, 367, 1924 March.
† M. N., 83, 493(1923).
terms “subordinate series” or “subordinate lines” to denote series or lines which are absorbed in transitions from one excited state to another excited state. In the usual spectroscopic terminology a principal series is one in which the constant term is derived from an orbit which is most conveniently described as having the azimuthal quantum number unity. Subordinate series are those whose constant terms are derived from orbits of higher azimuthal quantum numbers. It frequently happens that the state of lowest energy is one of azimuthal quantum number unity, and the two terminologies more or less coincide; this was in fact the case with all the atoms whose spectra were considered in our previous paper. But for ionised carbon, for example, the states of lowest energy are the pair $1\pi_1, 1\pi_2$, which give rise to the sharp and diffuse series, whilst the first $\sigma$ state, which gives rise to the “principal series” of optics and has the azimuthal quantum number unity, is one of greater energy. In stellar applications we require a special name for lines derived from the state of lowest energy, and we shall therefore use the word principal in the sense stated above. When we wish to refer to a “principal series” in the optical sense, we shall call it a “first principal series,” a “second principal series,” etc., as the case may be. A second principal series is necessarily a subordinate series in our terminology, since it is derived from the second level of azimuthal quantum number unity.

In a mass of gas in statistical equilibrium, the number of neutral atoms in the normal state steadily decreases as the temperature rises, owing to increasing ionisation. The number of neutral atoms in a given excited state increases to a maximum and then decreases. It follows that if the gas is producing absorption lines, the intensity of a principal line will steadily decrease as the temperature increases, but the intensity of a subordinate line will rise to a maximum and then decrease.

We showed in our previous paper that if $\chi_1$ is the ionisation potential of the atom, $\chi_1^{(r)}$ the (negative) energy of a given excited state, then for a given partial pressure $P_e$ of free electrons, the temperature $T$ of maximum concentration of atoms in the given excited state $\chi_1^{(r)}$ is given by

$$P_e = \frac{\chi_1^{(r)} + \frac{\hbar kT}{h\sigma_1}}{\chi_1^{(r)} - \chi_1^{(r)}} \cdot \frac{(2\pi m)^{\frac{3}{2}} (kT)^{\frac{3}{2}} h}{h^2 b_1(T)} \cdot e^{-\chi_1^{(r)} kT}.$$  \hspace{1cm} (1)

In (1) $m$ is the mass of the electron, $k$ is Boltzmann's constant, $h$ is Planck's constant, $\sigma_1$ is the symmetry number of the neutral atom, and $b_1(T)$ is the partition function, given by

$$b_1(T) = q_1 + q_1^{(2)} e^{-\chi_1^{(r)} kT} + q_1^{(3)} e^{-2\chi_1^{(r)} kT} + \ldots.$$ \hspace{1cm} (2)

In (2) the $q$'s are the weights of the corresponding stationary states, and the summation is extended over all the stationary states of which the given atom, in its neutral state, is capable. Equation (1) is an equation in $T$ whose root is $T_{\text{max}}$, but we found it convenient to assume values of $T_{\text{max}}$ and calculate from (1) the corresponding values of $P_e$. A certain difficulty connected with the convergence of the series in (2) has recently been removed by the work of H. C. Urey.* We return to this in § 9.