Parity Mixing in Nuclear Single-Particle Orbitals
and Direct Photonuclear Reactions.

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In a number of recent papers (1-4) single-particle wave functions (s.p.w.f.) with undefined parity have been proposed and used in nuclear many-body problems. Although this idea succeeded in explaining certain static nuclear properties (2-3), some limitations have been found in its application to nuclear reactions, such as elastic and quasi-free scattering of electrons (4). The aim of this note is to supplement the investigation in the case of direct photoreactions on odd-A nuclei at low energies, which are well known to be a sensitive test for s.p.w.f. (5-8).

In the usual approximation we describe the target nucleus to be composed by an inert core plus a valence nucleon. So, if s.p.w.f. with undefined parity are of the form (2)

\[ \psi_{njm} = \sum c_l(nj) \varphi_{njm} \]  

\[ (l = j \pm \frac{1}{2}) \]

where \( \varphi_{njm} \) is the normalized s.p.w.f. with good parity, and

\[ \sum_l |c_l(nj)|^2 = 1 \]

the total state function of the target nucleus in a simple shell model with \( j-j \) coupling.

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where \( \varepsilon_0 = \pm 1 \) refers to the parity of the state and \( \Psi_{\varepsilon_0}(j_{x\ldots x_{N-1}}(\alpha_1 J_1) j_y I_0 M_0) \) describes the vector coupling of the core (specified by the quantum numbers \( (\alpha_1 J_1) \)) to the odd (valence) \( N \)-th nucleon in the subshell \( j \).

General formulae for total cross-section, angular distribution and transverse polarization of the emitted photonucleon can be obtained in the Dirac's time-dependent perturbation theory \((6,10)\), already used in previous papers \((6,7)\), whose notation we follow here.

For the partial contributions to the differential cross-section we have

\[
\sigma_{ij}(X, \lambda, X', \lambda') = (2 - \delta_{iX,yX'}) \sum_{\lambda_0} \mathcal{K}(\varepsilon_0, I_0 J_0) \cos \left[ \frac{(\eta_i^{\lambda_0} - \eta_i^{\lambda_0'})}{2} (l - l' + \gamma) \right]
\]

\[
\cdot \left( j_{x_{N-1}}(\alpha_1 J_1) j_y I_0 \right) \left( j_{x_{N-1}}(\alpha_1 J_1') j_y I_0 \right) \mathcal{A}_{ij}^{(\lambda, \lambda')} (I_0 j_0 \xi) \pi(l' I J)
\]

\[
\cdot \mathcal{W}_{\lambda, \lambda'}^{(\xi)} (I_0 j_0 \xi) (|\lambda \lambda'h - h|J \xi) (j_{y_{N-1}} - \frac{1}{2}|J \xi) P_J (\cos \theta)
\]

where \( \xi \) represents the electromagnetic multipole transition \((\lambda = 1, 2 \text{ for } X = E, \lambda = 1 \text{ for } X = M)\); \( [\xi] \equiv [\alpha_1 J_1, I_0 J_0, l \alpha I, l' \alpha' I' \alpha'] \);

\[
\mathcal{K}(\varepsilon_0, I_0 J_0) = \delta_{i_0 j_0} [\xi_{i_0} (\eta_0 J_0)] \frac{1}{2} [1 + \varepsilon_0 (-1)^{i_0} \lambda_{i_0} (l, N) \lambda_{i_0} (l, N - 1)]
\]

where \( \lambda_{i_0} (l, N) \) is a normalization factor of the state function as defined in ref. \((7)\):

\[
\mathcal{W}_{\lambda, \lambda'}^{(\xi)} (I_0 j_0 \xi) = \mathcal{W}(j_{y_0} j_0 j I; J_1 \lambda) \mathcal{W}(j_{y_0} j_0 j' I'; J_1 \lambda') \mathcal{W}(I j_0 I' j'; J_1 J) \mathcal{W}(\lambda I \lambda' I'; J_0 J)
\]

and

\[
\mathcal{R}_{\lambda_0}^{(\xi_0)} = \mathcal{R}_{\lambda_0}^{(E)} + \mathcal{R}_{\lambda_0}^{(M)} = \int_0^\infty R_{\lambda_0}^{(E)} (r) R_{\lambda_0}^{(M)} (r) r^{2 + \lambda} dr
\]

is a radial matrix element (for \( X = E \), and the same with \( r^{2 + \lambda} \) replaced by \( r^{1 + \lambda} \) for \( X = M \)). Furthermore,

\[
\mathcal{A}_{ij}^{(E,E')} (I_0 j_0 \xi) = \frac{e^2}{4\pi} \left( \frac{\omega}{2 \sqrt{3 \lambda \sigma}} \right)^{\lambda + 1 - \lambda'} \left( \frac{j_{y_0}^{\lambda' + 1} j_{y_0}^{\lambda' + 1}}{2} \right) ^{\lambda - 1} \pi (l_{\xi}) (\lambda_{\xi} |l' \xi^{\lambda'}) (j_{y_0} |j_{y_0}^{\lambda' + 1} l_{\xi} |j_{y_0}^{\lambda' + 1} l_{\xi}^{\lambda'})
\]

\[
+ \delta_{i_0 j_0} \frac{1}{4\pi} M(l_{\xi} l_{\xi} j_{y_0} j_{y_0} j_{y_0} j_{y_0}) \delta_{i_0 j_0} [\xi_{i_0} (\eta_0 J_0)] \left( j_{y_0}^{\lambda'} j_{y_0}^{\lambda'} \right) ^{\lambda - 1} \pi (l_{\xi} |l' \xi^{\lambda'}) (j_{y_0} |j_{y_0}^{\lambda' + 1} l_{\xi} |j_{y_0}^{\lambda' + 1} l_{\xi}^{\lambda'})
\]

\((10)\) The calculations involve standard routine Racah algebra, well-known to specialists and usually omitted; to have an idea of intermediate steps, see: D. E. FREDERICK: Nucl. Phys., A 101, 250 (1967).