Electromagnetic Cavity Resonances in Accelerated Systems. - II

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Summary. — Electromagnetic-cavity resonances in a rotationally induced gravitational field are investigated using the gravitational tensor $g_{ab}$ as the structural element in Maxwell’s equations. The constitutive relations are expressed by tensorial relation in free space by $F^{kl} = V^{m} g_{2} g^{hi} g^{jk} F_{jk}$. The equations have the four-dimensional invariant form if $F_{jk}$ is considered as a covariant bivector in 4-dimensional space which admits all orthogonal homogeneous transformations (rotations and reflections). $G4$ is a subgroup of the general affine or Poincaré group. The method presented here is the metric approach generalized so as to accommodate all possible experimental data. This is achieved by simply decomposing the gravitational tensor into vacuum and material parts and performing proper transformations of the decomposed parts separately or simultaneously in accordance with the experimental configuration under consideration. Furthermore the comparison of this presentation with that of the nonmetric approach is discussed.

1. - Introduction.

In recent years a considerable effort has been invested in the understanding of electromagnetic resonances in accelerated systems. These investigations were mainly inspired by the greater experimental precision obtainable with the use of laser sources. A ring or closed-circuit laser, that sends counter rotating beams of light around a closed path provides the most sensitive gyro-

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scope yet invented. Theoretical investigations, however, have not been sufficiently general to cover all possible experimental configurations. The primary purpose of this paper is to close this gap in the metric method of approach of the classical electromagnetic theory.

Two of several approaches (1) which have been proposed are a relativistic (metric) approach and a nonmetric approach in which the gravitational tensor in the formalism is excluded and which treats the problem with greater generality. The advantages of the nonmetric approach have been discussed and rigorous solutions based on this type of formulation have been presented in a previous publication (hereafter to be called 1) (2).

The present paper has three purposes, namely: i) to bring into proper perspective the relation of the nonmetric to the metric approach by simply demonstrating that the metric approach is a special case of the nonmetric approach; ii) to enhance the understanding of the space-time transformation used in the solution of such a problem and to show that the relativistic approach renders no mathematical advantage by its heretofore avoidance of a thorough discussion of the nature of the space-time transformation appropriate to the problem; iii) to discuss the results in physical and geometrical optical solutions by decomposing the gravitational tensor and using proper transformations in accordance with experimental configurations.

2. – The Maxwell-Minkowski equations.

For the metric approach the gravitational tensor $g_{\alpha\beta}$ plays the role of a structural element of the Maxwell formulation of electromagnetic theory. Indeed the four-dimensional form of the electromagnetic equations in vacuo can be written in the following manner:

\begin{align}
(1) & \quad \partial_{[\mu} F_{\nu\lambda]} = 0, \\
(2) & \quad \partial_{\mu} \mathcal{F}^{\mu\nu} = -\mathcal{J}^{\nu}, \\
(3) & \quad \mathcal{F}^{\kappa\lambda} = \sqrt{\frac{\varepsilon_0}{\mu_0}} g^{\mu\nu} g^{\lambda\sigma} F_{\mu\nu},
\end{align}

where $F_{\mu\nu}$ is composed of $E$, and $B$, $\mathcal{F}^{\kappa\lambda}$ of $D$ and $H$, and $\mathcal{J}^{\nu}$ of $\varrho$ and $\mathcal{u}$. These equations have the four-dimensional invariant form in general rectilinear or curvilinear co-ordinates if $F_{\mu\nu}$ is considered as a covariant bivector in $G_4$. $\mathcal{F}^{\kappa\lambda}$

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