Singular Response of an Ideal Bose Gas.

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Summary. — The concept of singular response is defined and used for the investigation of the Bose-Einstein condensation of an ideal gas. It is shown that, apart from the well-known symmetry that is broken by the transition by the state with \( k = 0 \), also states with \( k \neq 0 \) participate in the condensation. The new condensed state is shown to possess off-diagonal long-range order and to be a product of Glauber coherent states with \( k = 0 \) and \( k \neq 0 \).

1. – Introduction.

We define singular response of the expectation value of an observable \( \mathcal{R} \) to an external probe \( \eta \) by

\[
\lim_{\eta \to 0} R(\eta) \neq R(0) .
\]

In words, we calculate \( R \) in the presence of \( \eta \), then let \( \eta \) go to zero. Next we calculate \( R \) with no external probe. If these two results are not the same we say that \( R \) responded in a singular way to the probe \( \eta \). Singular response can be used for the investigation of phase transitions. A simple example for this is the case where eq. (1) holds for all temperatures \( T \) lower than some critical temperature \( T_c \), while the response is nonsingular, i.e.

\[
\lim_{\eta \to 0} R(\eta) = R(0)
\]

for \( T > T_c \). Note that if the response is singular for all temperatures, it reflects the sensitivity of the system to the probe and the technique can be
used to obtain a dispersion relation for internal excitations of the system. The latter case is sometimes used (1) to obtain the plasma-frequency branch of the spectrum for electron-ion system. In general a singular response reflects intrinsic properties of the system on which the probe is applied.

In this paper we wish to study the Bose-Einstein condensation (BEC) from the point of view of singular response (SR). Preliminary results along this spirit were presented elsewhere (2). In Sect. 2 the early results are rederived in a slightly different way and we show the nonanalytic dependence of the chemical potential $\mu$ on the external probe as the latter is set equal to zero at the end of the calculations. The SR for our case of BEC is summarized in Sect. 3. In this Section one sees the reflection of the singular response in broken symmetry. This latter term means, in the present context, a special case of SR, viz. it implies the existence of a quantity $K$ that

$$\lim_{\eta \to 0} K(\eta) \neq 0,$$

while $K(0) = 0$. The results of the SR for the BEC are shown to imply the important consequence, which is true also for interacting bosons (3) namely, that for $T < T_c$ the system has a finite fraction of particles in a coherent state as defined by Glauber (4). (Note that everywhere we deal in the limit of particle number $N \to \infty$ while $N/V < \infty$ with $V$ the volume of our system.) The manner of the approach to zero of $\mu$ in BEC in our case is dealt with in more detail in Sect. 4 where we show that the SR of the free Bose particle implies that the condensation is of the form of a generalized Bose Einstein condensation (GBEC) as was introduced by Girardeau (5). It is shown that at the GBEC the system possesses an off-diagonal long-range order (ODLRO), a concept that was introduced by Yang (6) and discussed extensively since, in particular with its connection to superfluidity (7). The ODLRO of free bose system for $T > T_c$ was discussed earlier (8). In Sect. 5 we conclude and make contact between our discussion of the particular phase transition considered and the general discussion of phase transition as was given by Emch (9).

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