Electromagnetic-Wave Propagation in a Weakly Ionized Plasma. III.

O. De Barbieri, P. Franchi and A. Orefice

Istituto di Scienze Fisiche dell'Università - Milano
Gruppo del CNR per la Fisica dei Plasmi Debolmente Ionizzati - Milano

Summary. — This work contains the evaluation of the conductivity of a plasma with an arbitrary ionization ratio, in presence of a constant and uniform magnetic field. The calculation method is a generalization of the one employed in the preceding work.

1. — Introduction.

In two preceding works (1,2) (hereafter referred to as I and II respectively) the high-frequency conductivity of a plasma has been evaluated. In I the plasma is supposed to be weakly ionized and in presence of an external magnetic field \( \mathcal{H}_o \); the electronic distribution function \( f \) obeys Boltzmann's kinetic equation (eq. (3) of that paper) and a strong monochromatic wave of the type \( E(r, t) = E_o(r) \cos(\omega t + \varphi(r)) \) with \( \omega \approx \omega_n = e \mathcal{H}_o / mc \) propagates in the plasma. From the asymptotic solution of the Boltzmann equation an explicit form of the nonlinear high-frequency (complex) conductivity tensor of a weakly ionized plasma has been derived. Thus it was possible to specify under which conditions on the amplitude and frequency of the external electromagnetic field the usual formulas for the linear conductivity can be employed. However, it was not possible to give a reliable criterion for the applicability of the Lorentz gas model used in I.

This problem has been solved in (3), where a system of equations describing the dynamics of a many-species plasma in the presence of neutrals and with an arbitrary ionization ratio has been derived. A general formula for the linear high-frequency conductivity of a plasma has been evaluated in II by means of the aforementioned set of equations when there is no external magnetic field acting on the plasma.

This general formula embodies, in suitable limiting cases, the well-known expressions of the conductivity $\sigma$ for a Lorentz plasma (see I) and for a fully ionized plasma (4).

In the present work we want to extend the results of II to the important case of a plasma subjected to an external magnetic field $\mathcal{H}_0$.

The basic equations of this work are the obvious generalization of eqs. (21) and (22) of II (*), that is

\begin{align}
(1) \quad &\left(-i\omega + v_s(\mathbf{v}) + \frac{Z_s e}{m_sc} \mathbf{v} \times \mathcal{H}_0 \cdot \frac{\partial}{\partial \mathbf{v}}\right) f_s(\mathbf{v}, \omega) + \\
&-i\omega^2 Z_s \int \frac{4\pi k^{2-1}}{1 + k^2} P_s(\mathbf{k}, \mathbf{v}, \omega) = -\frac{Z_s e}{m_s} E_0 \cdot \frac{\partial F_s}{\partial \mathbf{v}},
\end{align}

\begin{align}
(2) \quad &\left[-i\omega + v - ik \cdot (\mathbf{v} - \mathbf{v}') + \frac{Z_s e}{m_s c} \mathbf{v} \times \mathcal{H}_0 \cdot \frac{\partial}{\partial \mathbf{v}} + \frac{Z_s e}{m_s c} \mathbf{v}' \times \mathcal{H}_0 \cdot \frac{\partial}{\partial \mathbf{v}'}\right] P_{sv}(\mathbf{k}, \mathbf{v}, \mathbf{v}', \omega) + \\
&+ i k^2 Z_s \omega_s^2 \left[\mathbf{k} \cdot \frac{\partial F_s'}{\partial \mathbf{v}'} P_{sv}(\mathbf{k}, \mathbf{v}, \omega) - ik^2 Z_s \omega_s^2 \mathbf{k} \cdot \frac{\partial F_s'}{\partial \mathbf{v}'} P_s(\mathbf{k}, \mathbf{v}, \omega)\right] = \\
&\left[2\pi \right]^3 \kappa_2 \kappa_k \left[\frac{1}{(2\pi)^3} \frac{1}{\kappa_2^2 + k^2} \left\{\mathbf{k} \cdot \left[ \frac{\partial}{\partial \mathbf{v}} - \frac{\partial}{\partial \mathbf{v}'} \right] [\mathbf{F}_s(\mathbf{v}) f_s(\mathbf{v}', \omega) + \mathbf{F}_s(\mathbf{v}') f_s(\mathbf{v}, \omega)\right] + \\
&+ i \frac{e}{\Theta} E_0 \cdot \left[ Z_s \omega_s \frac{\partial F_s}{\partial \mathbf{v}} \mathbf{F}_s(\mathbf{v}) + Z_s \omega_s^2 \frac{\partial F_s}{\partial \mathbf{v}'} \mathbf{F}_s(\mathbf{v}')\right]\right].
\end{align}

We admit that

\begin{equation}
(3) \quad \omega \simeq \omega_s \simeq v_s
\end{equation}

and we notice (4*) that if $\omega \tau_D >> 1$ (where $\tau_D$ is the cumulative 90° deflection time) the second term in the l.h.s of (1) is $O(1/\omega \tau_D)$ with respect to the other

- (3) O. De Barbieri and C. Maroli: On the dynamics of weakly ionized gases, accepted for publication by the Ann. of Phys.
- (*) Unless explicitly stated the symbols are the same as in II.