Heat Transfer in Relativistic Charged-Fluid Flow.

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Summary. — The problem of the dynamical effects of heat absorption or rejection on the flow of an electrically charged fluid such as an electron or ion gas is considered within the framework of special relativity. The source or sink for the heat transferred to or from the fluid may be imagined to be a heat reservoir whose velocity is in general not the same as the fluid velocity. The analysis of a relativistic Carnot cycle that operates between two heat reservoirs having relative motion with respect to each other provides a simple way to study the dynamical effects of heat transfer and to incorporate the moving heat reservoir model into the formalism of fluid dynamics. All the dynamical effects that are functions of the 4-gradient of the specific entropy of the fluid can be expressed in terms of an antisymmetric tensor, the entropy-force tensor, whose role in the equation of fluid motion is analogous to that of the electromagnetic field tensor.

1. — Introduction.

The familiar stress-energy tensor approach \(^{(1-2)}\) to relativistic fluid dynamics, which is summarized in Sect. 2, arrives at an equation of fluid motion in which the dynamical effects of heat flux in the fluid are given by the 4-divergence of a

\(^{(1)}\) Surveys of relativistic fluid dynamics can be found in most relativity textbooks. Of the available brief surveys, probably the most relevant to the present work is the one given in Chap. XV of Fluid Mechanics by L. D. Landau and E. M. Lifshitz (Reading, Mass., 1959). More detailed discussions are given by A. Lichnerowicz: Théories Relativistes de la Gravitation et de l'Electromagnetisme (Paris, 1955), Chap. IV-VI,
symmetric tensor \( Q^{ik} \) which is the contribution to the stress-energy tensor resulting from the presence of heat flux. Until more specific information concerning the form of \( Q^{ik} \) is provided, however, the analysis can be carried no further. This barrier can be circumvented by expressing the heat force term in the equation of motion directly in terms of a model that is both flexible enough to include most situations of physical interest, and simple enough to allow a direct insight into the processes producing the dynamical effects associated with heat transfer within the fluid. Such a model must also be related to the heat tensor \( Q^{ik} \) so that, given this tensor, the characteristic features of the model can be calculated. The intuitive thinking and calculation could, however, be carried out directly in terms of the model.

The moving heat reservoir model fulfills these requirements. The quantitative implementation of this model can best be carried out in terms of a relativistic Carnot cycle operating between two heat reservoirs having relative motion with respect to each other. This is done in Sect. 4. As an introduction to the relativistic Carnot cycle, a discussion is given in Sect. 3 of the Lorentz transformation of heat and temperature, which has been the subject of recent debate.

In Sect. 5 the moving heat reservoir model is incorporated into the equation of fluid motion, and the entropy-dependent contribution to the force is expressed in terms of the antisymmetric entropy force tensor which is analogous, in its dynamical effects, to the electromagnetic field tensor.

and by F. HALSWACHS: Théorie Relativiste des Fluides à Spín (Paris, 1960), Chap. IV, Sect. 2, and Appendix B.

The relativistic equations of fluid dynamics were first derived independently by G. HERGLOTZ: Ann. d. Phys., (4), 36, 493 (1911); E. LAMLA: Ann. d. Phys., (4), 37 772 (1912). Lamla’s equations were less general in that they were limited to the case of isentropic flow, whereas this restriction was not imposed in Herglotz’s derivation. Subsequent research has been limited almost exclusively either to isentropic flow (e.g. I. M. KHALATNIKOV: Zurn. Eksp. Teor. Fiz., 27, 529 (1954), a brief review of which is given in LANDAU and LIFSHITZ cited above, and A. H. TAUB: Phys. Rev., 103, 454 (1956)) or to the somewhat more general case of a barotropic fluid, i.e. one for which the pressure may be regarded as a function of the density alone (e.g. J. L. SYNGE: Proc. London Math. Soc., (2), 43, 376 (1937), and the work of LICHNEROWICZ, which is summarized in his book cited above). The papers cited in footnotes (2, 3) are not, however, restricted to either of these two special cases.

(2) C. ECKART: Phys. Rev., 58, 919 (1940). This paper concentrates on the thermodynamical aspects of relativistic fluid dynamics. Of all the fluid dynamical references cited, it and Khalatnikov’s paper are the most relevant to the present work. Both of these papers, like the present work, are within the framework of special, rather than general, relativity.

(3) O. PICHON: Ann. Inst. Poincaré, A 2, 21 (1965). This work, like that of SYNGE, LICHNEROWICZ and TAUB cited in footnote (2), is within the framework of general relativity. Its relevance to the present work lies in the fact that it presents alternatives to the treatment of viscosity and heat conduction in a charged fluid that differ from the approach taken in the present work.