Photon Escape Probabilities in Expanding Atmospheres

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Abstract. A comparison of mean number of scatterings and escape probabilities has been made in isotropic scattering and dipole scattering by using the angle-averaged partial frequency redistribution function \( R_I \). We have solved the equations of radiative transfer and statistical equilibrium simultaneously in a spherically symmetric expanding atmosphere. Two cases of atmospheric extension (i.e.) \( B/A = 3 \) and 10 (where \( B \) and \( A \) are the outer and inner radii of the atmosphere) have been treated.

We find that the partial frequency redistribution gives a larger mean number of scatterings compared to that given by complete redistribution. Velocities tend to reduce the mean number of scatterings and increase the mean escape probabilities.

Key words: mean number of scatterings—mean escape probability—partial redistribution function—isotropic scattering—dipole scattering

1. Introduction

It is well known that the radiative transfer effects in an optically thick resonance line are considerable and should be investigated by highly accurate methods. If the medium is optically thin, one can easily calculate, for example, the probability \( P_S \) that a photon, created at a given point in space and time, leaves the medium without being scattered or absorbed. If the optical depth at a normalized frequency \( x \) is given by \( \tau_x \), then \( P_S = \exp (-\tau_x) \). This relation is true in the case of certain subordinate lines. For resonance lines, \( \tau_x \) is much larger and a rigorous treatment of radiative transfer is required. Several people have studied the problem of mean number of scatterings and photon escape probability in a resonance line (Osterbrock 1962; Hummer 1964; Panagia and Ranieri 1973; Kunasz and Hummer 1974a, b) with various assumptions regarding geometry and nature of the media. However, in all these studies the main assumption was complete redistribution of photons in the line. A few have solved the radiative transfer equation correctly for
the resonance line in a moving media relative to the rest frame. In these calculations
the optical depths were assumed at line centre and mean number of scatterings and
the mean escape probabilities were computed. In a general case one must reduce
the number of free parameters such as optical depths, velocities etc. and obtain a
consistent solution of line transfer in the resonance lines. There exist now fast and
accurate techniques for obtaining simultaneous solutions of radiative transfer equa-
tion in a comoving frame and statistical equilibrium equation, for a resonance line
(Peraiah 1980). The aim of this paper is to use such calculations to compute the mean
number of scatterings and escape probabilities in a resonance line forming in a
spherically symmetric expanding atmosphere. The mean number of scatterings and
escape probabilities have been estimated with the partial redistribution function
$R_t$. Isotropic and dipole scattering functions have been employed for the sake of

2. Brief description of procedure and discussion of the results

The procedure of obtaining a simultaneous solution of radiative transfer equation
in a comoving frame and the statistical equilibrium equation for a non-LTE two-
level atom with complete redistribution is described in Peraiah (1980 henceforth
called Paper I). This procedure has been extended to include partial frequency
redistribution (Peraiah 1981). We have made use of the redistribution functions to
calculate the profile functions:

\[ \phi(x) = \int_{-\infty}^{\infty} R(x, x') \, dx', \]  \hspace{1cm} (1)

where

\[ R_{I-1}(x, x') = \frac{1}{\sqrt{\pi}} \int_{|x|}^{\infty} \exp(-t^2) \, dt \]  \hspace{1cm} (2)

for isotropic scattering (Unno 1952) and

\[ R_{I-2}(x, x') = \frac{3}{8} \left( \frac{1}{\sqrt{\pi}} \int_{|x|}^{\infty} \exp(-t^2) \, dt \right) \]  \hspace{1cm} (3)

\[ \times \left[ 3 + 2 (x^2 + x'^2) + 4 x^2 x'^2 \right] \]
\[ - \left( \frac{1}{\sqrt{\pi}} \exp(-|x|^2) \right) \left( \frac{1}{|x|^2} + 1 \right)^2, \]

for dipole scattering in the angle-averaged redistribution function. The statistical
equilibrium equation for a two-level atom that we have used is given by

\[ N_1 \left\{ B_{12} \int J_x \phi_x \, dx + C_{12} \right\} = N_2 \left\{ B_{21} \int J_x \phi_x \, dx + C_{21} + A_{a21} \right\}, \]  \hspace{1cm} (4)