Exchange-proofness or divorce-proofness?
Stability in one-sided matching markets

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Abstract

Two stability concepts for one-sided matching markets are analyzed: Gale-Shapley stability and \( \xi \)-stability. The first one applies best to markets where no status quo allocation is considered, whereas the second one is a solution to be used when property rights are allowed. A common problem of existence is shared by both solution concepts. Hence, we study economic environments where this problem does not exist, and present a family of agents’ preferences for which existence is guaranteed for both Gale-Shapley stable and \( \xi \)-stable outcomes.

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1. Introduction

This paper studies allocation problems for one-sided matching markets. These markets were introduced by Gale and Shapley [2] as the "roommate market

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They consider a set of students to be allocated in different rooms, in such a way that each one must share a room with another student. Given an allocation, the utility it produces to each student is uniquely determined by who her roommate is.

We can describe several other situations where the same type of allocation problem arises. The chess matching problem, in which some agents are matched for playing some chess matches; or the rural school problem, where some schools should be shared by rural populations, are additional examples for one-sided matching markets. Because of the great diversity of a problem qualified in this way, it is difficult to find a single suitable definition to describe what a “good allocation” should be for any of them.

The main solution concept analyzed by the literature on one-sided matching was introduced by Gale and Shapley [2]. These authors propose some properties to be satisfied by any allocation considered to be a satisfactory solution. The main solution they analyze is a stability property for the allocations. They propose a concept of stability involving an idea of collective rationality. We will refer to this solution as GS-stability. It is a solution also proposed for marriage problems, which is why we say that GS-stability is a “divorce-proof” based solution.

The central hypothesis for this condition is that any pair of agents can become roommates if they both wish to. We consider this hypothesis to be satisfactory only when no allocation provides the agents with property rights. Yet, there are one-sided matching markets where each allocation gives some property rights to the agents. Then, the actions of any pair of agents may have an impact on third parties, and these may have a say if their property rights are affected. Since GS-stability does not allow for such considerations, we challenge its general usefulness and explore an alternative solution concept. ξ-stability should be useful in describing a collective rationality idea for contexts where property rights are allowed. Allocations satisfying this requirement will conform to a notion of “exchange-proofness” within markets.

This concept is not intended as an alternative to the GS-stability concept, but rather as something which complements it so that it will be suitable to cover those types of one-sided matching markets where GS-stability does not fare well. That is, given any one-sided matching market, any satisfactory allocation should be either ξ-stable if it gives property rights to the agents or GS-stable if it does not.

Both solution concepts present some good properties, but also share a common problem. Given a one-sided matching market, the set of its GS-stable outcomes as well as the set of its ξ-stable allocations may well be empty. The direction we explore to avoid this problem is employed extensively in Economic Theory. We look for sufficient conditions in economic environments under which positive results to stability problems become possible. We will present classes of individual

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Roommate markets and one-sided matchings are terms used interchangeably throughout the paper.