Reduced-Amplitude Equations
for Two- and Three-Particle Systems - II (').

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Summary. — Reduced amplitudes are simultaneously defined for two particle→three particle and three particle→three particle processes. They are shown to lack the physical-region singularities: the subenergy cuts, the total discontinuity, the separate energy cuts and, in 3→3, the cross-energy poles. A reduced amplitude for two particle→two particle scattering lacking the three-particle, as well as the two-particle, cut is also defined. This simultaneously involves the three particle→two particle interaction for which a reduced amplitude is thus obtained. This analysis is relativistic and nonpotential-theoretic.

1. — Introduction.

The general introduction to this work and its motivation are given in paper I (1), which also includes a discussion of the significance and interpretation of the final equations and gives two deductions as special cases. It is the purpose to prove in this paper the main results: the definition of the reduced amplitudes, free of the physical-region singularities and the reduction of the equations to the final form used in paper I. It was thought convenient to present the work

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in two separate parts because of the rather involved algebraic nature of the main derivations given in the present paper.

The theory of the two-particle «K-matrix» has recently been extended by Branson (2) to three-particle scattering under the assumption that the quantum numbers of the scattering particles forbid a two-particle intermediate state. It is with the aim of relaxing this assumption that in Sect. 2, using different methods, we simultaneously define two reduced amplitudes \( K^{23} \) and \( K^{33} \) for two-particle \( \rightarrow \) three-particle and three-particle \( \rightarrow \) three-particle interactions respectively. The transition amplitude \( A^{23} \) (for 3 \( \rightarrow \) 3 scattering) cannot be considered separately when this assumption is dropped. The amplitudes \( K^{23} \) and \( K^{33} \) are defined in terms of \( A^{22} \) and \( A^{33} \) (simultaneously) and a number of auxiliary functions denoted \( R^{23}, S^{23}, M^{33} \) and \( N^{33} \). Conditions are placed on these auxiliary functions by requiring \( K^{23} \) and \( K^{33} \) to be free of the subenergy cuts and the total discontinuity below the four-particle threshold. The equations arrived at (for the auxiliary functions) are then shown to provide sufficient conditions for \( K^{23} \) and \( K^{33} \) to lack the singularities. \( K^{23} \) and \( K^{33} \) are also shown to lack the energy cuts separately. When \( K^{23} = 0 \), the reduced amplitude \( K^{33} \) becomes identical with the amplitude \( G \) of Branson (2).

In Sect. 3 we extend the «K-matrix» equation for two-particle \( \rightarrow \) two-particle scattering by defining a reduced amplitude \( H^{22} \) that lacks the three-particle cut in \( A^{22} \), as well as the two-particle cut. This simultaneously involves the amplitude \( A^{22} \) for which a reduced amplitude \( K^{22} \) is thus defined. We find that some of the auxiliary functions we used in Sect. 2 turn up again in the definition of \( H^{22} \) and \( K^{22} \).

The complete set of equations (twenty-six in all) for the two-particle \( \rightarrow \) three-particle and three-particle \( \rightarrow \) three-particle processes in Sect. 2, like Branson's equations (2), contains much redundancy. This was useful (in Sect. 2) in providing some insight in the algebraic manipulations since it rendered term-by-term comparison with the unitarity equations possible. In Sect. 4 the reduction of the equations to a proper mathematical form (eight equations) is performed. When the \( K \)-functions are assumed, one has to solve three coupled integral equations to obtain either \( A^3 \) or \( A^{33} \). In Sect. 4 we also include the specialization of the reduced equations to the case of identical particles.

2. – The reduced amplitudes \( K^{23}, K^{33} \).

It is the purpose in this Section to define simultaneously reduced amplitudes \( K^{23}, K^{33} \) for the transition amplitudes \( A^{23}, A^{33} \) such that they lack normal-threshold singularities in the physical region. We shall adopt Branson's propa-