The Geometry of the $SL_{2,\mathbb{C}}$ Gauge Formulation of General Relativity.

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Summary. — The formulation of Einstein's general theory of relativity as an $SL_{2,\mathbb{C}}$ gauge theory is considered. Use is made of the language of fibre bundles and general arguments are put forward in favour of the $SL_{2,\mathbb{C}}$ approach to problems connected with the study of the space-time structure. The possibility of deriving the dynamics of the theory from a Yang-Mills-type Lagrangian density is discussed. Finally, the spinor approach is compared with other approaches to the problem of formulating Einstein's theory as a gauge theory.

1. — Introduction.

Ever since EINSTEIN put forward his general theory of relativity (1-3), there have been many attempts at formulating a unified theory of gravitation and other interactions based on geometrically biased generalizations of Einstein's theory (4-7). Despite the fact that most of these attempts at unification are justifiable in one way or another, none of them prophesy any new measurable non-Newtonian effects and, furthermore, they have failed to lead to any significant physical insight in the theory of interactions and they have not influenced the present line of scientific thinking.

The general theory of relativity is not merely a relativistic theory of gravitation, but is also a theory of the geometry of space-time and one of the most fundamental questions today is the role of the space-time structure in the world

of microphysics. The theory of elementary-particle physics is developed in Minkowski space-time, a given universal classical background space-time structure with the particles defined on it. This is in complete contrast to the general theory of relativity in which the components of the metric tensor, the quantities which define the geometry, are the dynamical fields.

Apart from the question of unification of Einstein’s theory with other theories, there is another problem intricately related to the investigation of the space-time structure and that is the quantization of the gravitational field. This problem has been well reviewed and discussed in the literature \(^{(8-12)}\) and will not be dealt with here. We shall, however, remind the reader that the conventional methods used for quantizing gravity are full of difficulties and unsolved problems. It is hoped that the formulation of Einstein’s theory of general relativity, as presented here, will offer a practical and effective alternative to these approaches. An initial step in this direction was recently made by Wódkiewicz \(^{(11)}\), who, using the gravitational-field variables defined in this article, quantized gravity employing Mandelstam’s path-dependent formulation of quantum electrodynamics \(^{(15)}\).

The theory propounded in this work was first proposed by Carmeli \(^{(16)}\) and subsequently developed by Carmeli and Carmeli and co-workers \(^{(17-32)}\).


