Mobility of Charged Impurities in Neutral Liquids (*) (**).

R. Lobo

Departamento de Física e Ciência dos Materiais
Instituto de Física e Química de S. Carlos, USP - São Carlos

(ricevuto il 19 Giugno 1973; manoscritto revisionato ricevuto il 29 Settembre 1973)

Summary. — The theory of the mobility of charged impurities in neutral liquids is developed from a few phenomenological parameters, in terms of the structure factors for both liquid and impurities.

1. — Introduction.

In recent years, the theory of the mobility of charged impurities in neutral liquids has been the subject of several papers.

The purpose of this work is to derive the expression for the mobility in the limit of small fields without any assumption on the kind of liquid, beyond that of being neutral, or on the kind of impurities present. The hydrodynamic mass of the impurities should be calculated as in ref. (1), for instance, and corresponds to the whole self-energy term on the left-hand side of the Boltzmann equation (2). We shall be dealing with the collision term. No calculation for the collision contribution to the self-energy term is attempted in this paper.

The only functions defining our system are the two structure factors $S_L$ and $S_I$ (from here on $L$ and $I$ are the subscripts corresponding to liquid and impurity respectively), the densities, the liquid-impurity scattering potential $\Phi$ (*) and the relaxation time for the liquid $\tau_L$ due to collisions with the walls.

(*) The speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(**) Work supported in part by BNDE, CAPES, CNPq and FAPESP.


(3) This potential may be replaced by the scattering matrix $T$. 
or with other imperfections. \( \tau_L \) is introduced in a phenomenological way. It has been calculated in the literature for some liquids such as \(^3\)He, \(^4\)He and \(^3\)He-\(^4\)He mixtures, as a function of temperature (\(^5\)).

The results obtained in this paper can be reduced under certain approximations to most of the work in the literature, but go beyond, by introducing the actual structure factors of both liquid and impurities, instead of the effective-mass approximation, so that first sound, zero sound in \(^3\)He, rotons and phonons in \(^4\)He are naturally introduced.

The drag of the host liquid is also taken into account from the beginning and, in the limit \( \tau_L \rightarrow \infty \), the impurity mobility goes to infinity, as it should, since in this limit the liquid would be moving along with the impurities.

2. - Calculation of the impurity mobility.

The calculation is reduced to the solution of the equations relating the change in momenta of the liquid and the impurities as coupled systems.

In equilibrium the change in momentum of the whole system

\[
\hat{P}_L = \hat{P}_{L \rightarrow L} + \hat{P}_{L \rightarrow \nu} + \hat{P}_p + \hat{P}_n
\]

equals zero. \( \hat{P}_{L \rightarrow L} \) is the change of the liquid momentum due to the collisions with the impurities, \( \hat{P}_p \) is the change of the impurity momentum due to external field \( \epsilon \) and \( \hat{P}_n \) is the change due to the relaxation of the liquid momentum.

The main problem here is to evaluate \( \hat{P}_{L \rightarrow L} = - \hat{P}_{L \rightarrow \nu} \). To obtain \( \hat{P}_{L \rightarrow L} \) we use the golden rule in the context of quantum statistical mechanics

\[
\hat{P}_{L \rightarrow L} = \frac{1}{2} \sum_q 2\pi |\Phi(q)|^2 \langle q | \hat{q} \rangle \exp \left[ -\beta E_n \right],
\]

where we put \( \hbar = 1 \), and \( \hat{q}(q) \) is the Fourier transform of the density operator, \( \beta = 1/k_B T \), where \( k_B \) is the Boltzmann constant, \( T \) the absolute temperature and \( Z \) is the canonical partition function.

To evaluate the matrix elements, we make the Hartree approximation, i.e. we assume that the wave functions for the impurities and liquid can be taken as product wave functions. (Mass renormalizations have already been