K^0 \rightarrow \mu^+ + \mu^- as a Test of Neutral Lepton Currents (*)

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The decay K^0 \rightarrow \mu^+ + \mu^- is forbidden in first order by the usual weak interaction Hamiltonian, which contains no neutral lepton currents. However, a coupling of K^0 to a neutral lepton current can be induced electromagnetically. The object of this note is to give an estimate of the rate of the above decay arising by such an induced coupling. We make use of the recently reported (*) rate for K^0 \rightarrow \gamma + \gamma, which permits us to make a more firm estimate of K^0 \rightarrow \mu^+ + \mu^- than previously possible (2).

The Feynman diagram for the process is shown in Fig. 1. The K\gamma\gamma coupling (Lorentz-invariant, gauge-invariant and CP-invariant (3)) is given by

\[ \frac{f_{K\gamma\gamma}}{m_k} \epsilon_{\mu\nu\alpha\beta} \epsilon'_{\mu\nu\alpha\beta} k_\mu k'_{\mu} \]

where \( \epsilon, \epsilon' \) are the polarization vectors, and \( k, l \) the four-momenta of the two photons. \( f_{K\gamma\gamma} \) is a dimensionless coupling constant, defined by

\[ \text{Rate} (K^0 \rightarrow \gamma + \gamma) = \frac{|f_{K\gamma\gamma}|^2}{64\pi} m_k \]

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(3) The assumption of CP-invariance at the K\gamma\gamma vertex is not crucial to the calculation of the rate of K^0 \rightarrow \mu^+ + \mu^-.
$m_K$ denotes the mass of the $K^0$. The invariant amplitude is given by

$$F = \frac{f_{K\gamma\gamma}}{m_K} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \left( \frac{1}{(p - k)^2 + i\epsilon} \varepsilon_{\mu\nu\sigma\kappa}(p - k)_{\sigma} \right) \cdot \bar{u}(p_-) \left( \gamma_\mu - \gamma_\nu p_+ + \gamma_\kappa k - m + i\epsilon \gamma_\nu \right) u(p_+).$$

Here $p_+$ ($p_-$) are the momenta of $\mu^+$ ($\mu^-$), and $m$ the muon mass. In the rest frame of the $K^0$ meson, $p = (m_K, 0)$, the integral simplifies to

$$F = 2f_{K\gamma\gamma} e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \frac{1}{(p - k)^2 + i\epsilon} \frac{k^2}{k^2 - 2p_+ k + i\epsilon},$$

where the spinors have been chosen to correspond to $\mu^+$ and $\mu^-$ emerging with the same helicity.

The integral in eq. (4) diverges logarithmically, and must be rendered convergent by introducing a cut-off that describes the form factor of the $K_{\gamma\gamma}$ vertex. However, the imaginary part of the integral is finite and cut-off-independent, and we evaluate it separately. This is done by making the replacement

$$\frac{1}{k^2 + i\epsilon} \frac{1}{(p - k)^2 + i\epsilon} \rightarrow 2\pi^2 \delta(k^2) \delta((p - k)^2)$$

in accordance with the Cutkosky rule. Equivalently, one may apply unitarity to find the imaginary part of $\langle \mu^+ \mu^- | T | K^0 \rangle$ (4), retaining only the two-photon intermediate state, and calculating the amplitude $\langle \mu^+ \mu^- | T | \gamma\gamma' \rangle$ in the Born approximation. Either way, we obtain

$$\text{Im} \ F = 4\pi^2 f_{K\gamma\gamma} e^2 \int \frac{d^4k}{(2\pi)^4} \delta(k^2) \delta((p - k)^2) \frac{k^2}{k^2 - 2p_+ k} =$$

$$= \frac{1}{32\pi} f_{K\gamma\gamma} e^2 \left( \frac{m_K}{m_K^2 - 4m^2} \right)^1 \log \frac{m_K + (m_K^2 - 4m^2)^{\frac{1}{2}}}{m_K - (m_K^2 - 4m^2)^{\frac{1}{2}}} = f_{K\gamma\gamma} \alpha(0.42),$$

where $\alpha = e^2/4\pi = 1/137$.

To evaluate the real part, we introduce a convergence factor $\Lambda^2/(A^2 + k^2)$ in the integrand of eq. (4). The Gell-Mann–Sharp–Wagner model suggests that $\Lambda$ should be of the order of a vector-meson mass. Denoting the amplitude after cut-off by $F^{(c)}$, we get

$$F^{(c)} = 2f_{K\gamma\gamma} e^2 \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 + A^2 + i\epsilon} \right] \frac{1}{(p - k)^2 + i\epsilon} \frac{k^2}{k^2 - 2p_+ k}.$$

In Table I, we give some numerical estimates of the real part of $F^{(c)}$ for different values of the cut-off. Combining these estimates with the imaginary part given in


(1)