The Classical Motion of an Extended Charged Particle Revisited.

I. DE LA PEÑA

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**Summary.** — The motion of a nonrelativistic extended self-interacting particle is analysed. The equation of motion is integro-differential and generates, at variance with the pointlike case, a strictly causal behaviour, thus overcoming all the fundamental shortcomings of the Abraham-Lorentz theory. The motion is endowed with memory, which generates effects totally absent in the structureless case, such as the existence of characteristic damped oscillations, whose frequency and number are determined by the specific structure.

1. - Introduction.

The classical movement of an electrically charged particle is commonly described by an equation due to ABRAHAM and LORENTZ (1),

(1) \[ m\ddot{r} = F_{\text{ext}} + m\gamma \dot{r}, \]

where self-interaction is taken into account by considering the mass \( m \) of the particle to be the sum of an inertial mass and of an electromagnetic

contribution and by including the radiation reaction \( m \tau \vec{F} \) which contains the characteristic time

\[
\tau = \frac{2e^2}{3mc^2}.
\]

This is, except for a geometric factor \( \frac{2}{3} \), the time needed for light to traverse the particle classical radius \( e^2/mc^2 \). Equation (1) has, for a time-dependent external force \( F_{\text{ext}}(t) \) applied at \( t = 0 \), the general solution

\[
\vec{r}(t) = \exp[t/\tau]\left[\vec{r}(0) - \frac{1}{m\tau} \int_0^t \exp[-t'/\tau] F_{\text{ext}}(t') \, dt'\right].
\]

But eq. (1) and its solution have well-known drawbacks, which we need not do more than list here (24):

i) The Abraham-Lorentz equation is only approximate and does not apply at all to the free particle (4).

ii) For a point charge the electromagnetic mass diverges.

iii) The electromagnetic contribution to the mass appears with an odd factor \( \frac{4}{3} \). For later convenience, we present here a brief account of the origin of this difficulty. The Abraham-Lorentz equation is derived from the assumption that the electromagnetic moment contained within the (extended) particle is given by

\[
P = \int \frac{S}{c^2} \, d^3r,
\]

where \( S \) is the Poynting vector. However, the relativistic version of this equation, namely

\[
p_{\mu} = \frac{i}{c} \int T_{\mu\nu} \, d^3r,
\]

where \( T_{\mu\nu} \) is the stress-energy tensor of the self-field, is not Lorentz covariant;

(3) J. L. Jiménez and O. L. Fuchs: The integrodifferential version of the Abraham-Lorentz equation, preprint OFIN 19-80, Facultad de Ciencias, UNAM (to be published). This is a recent review of the subject.