Stationary Cosmological Solution with Torsion.

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Summary. — The role of torsion and spin has been studied in the case of stationary universe with Weyssenhoff fluid under rigid rotation. It is shown that, when the pressure bears a linear relation with the density, there exist closed timelike lines. Only when the space-time is $S - T$ homogeneous, the spin can avoid closed timelike lines.

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1. — Introduction.

The spatially open, rigidly rotating stationary universe with closed timelike lines presented by Godel (1) in the late forty found renewed interest in the sixties. Although it was devoid of any physical relevance with observational cosmology, its rich geometrical structures gave an impetus for investigation of rotating universes. Oszvath and Schucking (2) exhibited a homogeneous universe which has no closed timelike lines. The rotation of the Universe, however, is not rigid and the cosmological term has to be retained. An inhomogeneous solution with rigid rotation given long ago by Van Stockum (3) also contains closed timelike lines. The nonhomogeneous solution which is stationary obtained by Maitra (4) has no closed timelike lines, but the rotation

is again nonrigid. Thus one finds that for stationary universes the problem of closed timelike lines is inseparably linked with the rigid rotation, no matter whether the space-time is spatially homogeneous or not and if the matter content of the universe is a perfect fluid with or without any arcane combination of other fields.

Recently Vasconcellos Vaidya et al. (5) studied the Godel-type metrics using the energy-momentum tensor of a spinning fluid obtained by Ray and Smallley (6) in the Einstein theory from the variational principle. Their solution contains no closed timelike lines.

The purpose of this paper is to study the role of spin in the stationary universes in the Einstein-Cartan theory. The attractive feature of the EC theory is that it provides a geometrical framework for the description of spin. Ray and Smallley (7) presented recently an improved perfect-fluid energy-momentum tensor with spin in the EC space-time. They used a Lagrangian variational principle based upon the tetrad formalism of Halbwach and the method of constraints of Ray (8). The first exact solution of the EC equations in which the spin density of the source manifests itself by a global effect, inducing magneticlike components in the gravitational field outside an infinite static cylinder with a net spin polarization along its axis of symmetry, was obtained by Tsoubelis (9). It provides a concrete theoretical model to test the EC theory by an Aharonov-Bohm-type experiment suggested by Stachel.

2. - Einstein-Cartan equations.

The Einstein-Cartan equations in tetradic form read

\begin{align}
R^A_{\ B} - \frac{1}{2} R \delta^A_{\ B} &= \kappa \epsilon^A_{\ B}, \\
Q^{A}_{\ BC} - \delta^A_{\ D} Q^{D}_{\ BC} - \delta^A_{\ E} Q^{D}_{\ BD} &= \kappa S^{A}_{\ BC},
\end{align}

where \( \epsilon^A_{\ B} \) is the canonical energy-momentum tensor, \( Q^{A}_{\ BC} \) the torsion tensor, \( S^{A}_{\ BC} \) the spin density tensor and \( \kappa = 8\pi G/c^4 \). The spin density tensor is defined according to the classical description of spin by the relation

\begin{equation}
S^{A}_{\ BC} = \nu^A S_{\ BC} \text{ with } \nu^B S_{\ BC} = 0,
\end{equation}