Dirac Equation and Energy Levels of Hydrogenlike Atoms in Robertson-Walker Metric.

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Summary. — The Dirac equation of the hydrogenlike atom in the Robertson-Walker metric is studied in this paper by means of tetrad formalism. The decoupled Dirac equation is derived and energy levels of hydrogen atoms taking into account the cosmological radius are obtained by using the perturbation expansion.

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1. — Introduction.

The influence of cosmological effects on the energy levels of the hydrogen atom in the Dirac equation has been a subject of interest and discussion for a long time (1,2). In (2) it is suggested that the spectral line of the hydrogen atom might be changed with distance if $e^2/hc$ is not constant. Recently

Parker (3) has used flat-space-time solutions of the Schrödinger and Dirac equations as a basis to calculate curvature contributions to the energy levels by first-order perturbation theory. Bessis and his co-operators got the solution of the nonrelativistic wave equation in a space of constant curvature (4) and the curved form of the one-electron fine-structure Hamiltonian or the curved form of the relativistic correction of the kinetic energy (5) by use of a ladder operator technique. But these authors have not solved the problem straightforward from Dirac equation. In this paper, the Dirac equation of the hydrogen-like atom is discussed in the Robertson-Walker metric of \( K = \pm 1, 0, -1 \) in terms of a spin coefficient method. The separated equations are obtained. The energy levels of the hydrogen-like atom (including relativistic effects) are calculated by using the perturbation expansion, by taking into account also the influence of the Universe on the hydrogen lines. But this influence is too small even for the most modern device to observe. In the early universe the temperature was so high that no atoms existed. Thus, even though the radius of the Universe was small and would, therefore, have had a large influence on hydrogen, had this existed, it was only after the recombination epoch that the Universe became transparent and the consideration of the hydrogen spectrum lines is meaningful.

From this time on, however, the radius of the Universe is larger than \( 5 \cdot 10^4 \, \text{ly} \), which has only a negligible influence on the hydrogen lines. Despite of this, we obtain in this paper the separated Dirac equation in the Robertson-Walker metric and make an approximate calculation of the energy levels of the hydrogen atom as affected by the radius of the Universe. These energy levels coincide with those of the normal solution of the Dirac equation when the radius of the Universe goes to infinity.

2. – The null tetrad and the spin coefficients of the Robertson-Walker metric.

The Robertson-Walker metric in Gaussian normal co-ordinates \((t, \chi, \theta, \varphi)\), comoving with the dust particles, is given by

\[
ds^2 = dt^2 - a'(t)(d\chi^2 + s_k^2 d\Omega^2),
\]

where \( \Omega \) is the solid angle and \( s_k = K^{-1} \sin K \chi \) and the values 1, 0 and \(-1\) of \( K \) correspond, respectively, to closed, flat and open cosmological models. Furthermore, we define

\[
dt = a'(t) \, d\tau = a(t) \, d\tau,
\]