Continuous and Discrete Matrix Burgers' Hierarchies.

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Summary. — We derive two hierarchies of matrix nonlinear evolution equations which reduce to the Burgers' hierarchy in the scalar case and can be linearized by a matrix analogue of the Hopf-Cole transformation. For these hierarchies we display the associated class of Bäcklund transformations and show some special kinds of explicit solutions. Moreover, by exploiting a discrete version of the Hopf-Cole transformation, we are also able to construct two hierarchies of linearizable nonlinear difference evolution equations and to derive for them Bäcklund transformations and explicit solutions.

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1. - Introduction.

It is well known (1) that Burgers' equation

\[ u_t = u u_{xx} - u u_x \]  \hspace{1cm} (1.1)

can be linearized through the Hopf-Cole transformation

\[ u = - 2\nu \varphi \varphi_x \]  \hspace{1cm} (1.2)

which maps it into the heat equation

\begin{equation}
\varphi_t = \nu \varphi_{xx}.
\end{equation}

More recently (2), it has been proved that Burgers' equation is a member of a whole hierarchy of nonlinear evolution equations (NEEs) which can be linearized by the same Hopf-Cole transformation. It seems thus worthwhile looking for a matrix generalization of this hierarchy, i.e. for a class of non-Abelian NEEs which can be linearized by a Hopf-Cole-type transformation.

In this work we exhibit two classes of matrix NEEs, both reducing to Burgers' hierarchy in the scalar case, and two classes of matrix nonlinear difference evolution equations (NDEEs) which can be linearized by suitable analogues of the Hopf-Cole transformation.

At a first glance one can realize that these classes contain some physically interesting equations, as, for instance, a dissipative version of the N-wave equation in the continuous case and a matrix Volterra-type equation in the discrete case. We postpone to a subsequent paper a systematic investigation of all possible interesting reductions. For all these hierarchies of evolution equations we are able to construct the corresponding classes of Bäcklund transformations (BTs) which allow us to obtain different ladders of explicit solutions: shock-type solutions, soliton-type solutions and rational solutions.

In sect. 2 we display the two classes of NEEs together with their linearization. In sect. 3 we derive an extension of the classes given in the previous section, so as to include terms with linearly x-dependent coefficients. The corresponding BTs and the nonlinear superposition principle (NSP) are obtained in sect. 4, while sect. 5 is devoted to the study of the discrete matrix Burgers' hierarchy, together with the associated BTs and NSP.

Finally in sect. 6 we solve the simplest nontrivial BTs, both for the continuous and for the discrete cases, and thus derive shock-type solutions, soliton-type solutions and rational solutions.

2. - Matrix NEEs.

Let us consider the linear problem

\begin{equation}
L \varphi = 0,
\end{equation}

\begin{equation}
L = \partial_x - U,
\end{equation}