Stochastic-Time Description of Transitions in Unstable and Multistable Systems (*).

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Summary. — The decay of a macroscopic unstable state implies anomalous fluctuations in the amplitudes of the decaying parameters, which are the transient extension of the stationary divergences at the critical point of phase transitions. These decays are best studied, theoretically and experimentally, via the stochastic times of intersection of a given threshold. Besides yielding a closed solvable set of moment equations, the stochastic-time approach permits to discriminate the transient fluctuations due to the spread in the initial conditions from those arising from noise along the path. These latter ones limit the validity of the so-called asymptotic approximation. Here we develop a detailed theory including scaling laws and then compare it with experimental measurements in order to show the limit of previous approaches.

1. – Introduction.

A macroscopic system displays a collective or co-operative behaviour whenever at least one of the observables (order parameter) has long-range space correlations within the system. For systems in thermal equilibrium, the physics of phase transitions has nowadays been set within the general framework of the renormalization group theory. In particular, when the interparticle forces are long range, the Landau mean-field theory is sufficient to describe the transitions.

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In open systems, that is away from thermal equilibrium, similar long-range correlated fluctuations may appear. Their behaviour depends on the actions exerted at the boundaries, that is on one or many control parameters assigned at the border rather than distributed in the medium.

The stationary solutions for the open-system dynamics may offer strong analogies with the equilibrium phase transitions. Usually these changes have been studied by a slow setting of the external parameter, in order to measure the stationary fluctuations and their associated spectra around each equilibrium point. A classification of these transitions is under way (1), by an extensive use of sophisticated measurements (2) or computer experiments (3). In particular, in quantum optics one can perform accurate statistical measurements by photon counting statistics; this method has been used to explore quantum-optical transitions (4).

More dramatic evidence, as well as detailed information, on the decay of an unstable state and its leading to multiple nearby stable positions with different branching probabilities, can be obtained by applying sudden jumps to the driving parameter and observing the statistical transients (5). These should by no means be compared with stationary time correlations (or their frequency spectra), since a linear regression is no longer valid, with the non-linearities playing their full role. Furthermore, when the system is prepared in an unstable state, no net systematic forces are applied on its observables and the decay is initiated by microscopic fluctuations. In the first linear part of the decay process the fluctuations are amplified; hence, during the transient and until nonlinear saturation near the new stable point reduces them, fluctuations do not scale with the reciprocal of the system size, as it is at equilibrium.

A simple-minded explanation of the anomalous fluctuations can be made with reference to the model of fig. 1.


