On Time Delay in the $S$-Wave Scattering.

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Summary. -- We have found that the time delay of the $S$-wave zero-energy resonance scattering must be expressed as the sum of the ordinary term $2(d\delta_0/dE)$ and the new term $2/F$. Where $F$ denotes the half-width of wave packet, and $d\delta_0/dE$ does the derivative of the $S$-wave phase shift with respect to energy. The main aim of this paper is to describe the analytical derivation of the existence of the term $2/F$.

It is well known that the $S$-wave state, different from the other states, has no resonance width for zero-energy scattering due to the lack of the centrifugal barrier (1-3). So it is an interesting problem how the time duration of the resonance scattering in this state is given in relation to the Eisenbut and Wigner's notion (3-4). Extended discussions for their notion have been done by many authors and the time delay of the $l$-th wave scattering has been inferred as the energy derivative of the phase shift $2(d\delta_0/dE)$ (5-9). However, in the case of the $S$-wave resonance scattering,

(5) For a review of this subject see, for example, M. L. Goldberger and K. M. Watson: Collision Theory (Wiley, New York, N. Y., 1964), Chapt. 8.
we will argue a modification is necessary. In the following we show that a new term must be added to the ordinary term for the time delay of this case (10-13).

We shall be dealing with the two-body collision time delay in the elastic scattering, where the system is represented by the two operators $H_0$ and $H = H_0 + V$, and $H_0 = (1/2M)P^2 + (1/2\mu)p^2$ denotes the kinetic energy of the particles and $V$ the interaction which produces the scattering. For simplicity, we assume that the interaction is a central short-range potential $V(r)$. Our formulation is based on the one in ref. (3).

Let $\Phi(t)$ be the two-body wave packet for the Hamiltonian without interaction. The time evolution of this packet may be written with $\hbar = 1$ as

$$(1) \quad \Phi(t) = \int d\mathbf{Q} \, \mathcal{A}(\mathbf{Q} - \mathbf{P})(2\pi)^{-\frac{3}{2}} \exp \{i(Q \cdot R - E_0 t)\} \int \! d\mathbf{q} \, a(q, p) \psi_\mathbf{q}(r) \exp \{-it_\mathbf{q} t\},$$

where variables $\mathbf{R}$, $\mathbf{Q}$ and $E_0$ are the spatial co-ordinate, momentum and energy in the centre-of-mass co-ordinate system, respectively. The corresponding quantities in the barycentric co-ordinate system are $\mathbf{r}$, $\mathbf{q}$ and $\epsilon_\mathbf{q}$. The $\psi_\mathbf{q}(r)$ denotes the relative motion and satisfies the following Schrödinger equation:

$$(2) \quad -(1/2\mu) \nabla_r^2 \psi_\mathbf{q}(r) = \epsilon_\mathbf{q} \psi_\mathbf{q}(r)$$

with

$$(3) \quad \psi_\mathbf{q}(r) = (2\pi)^{-\frac{3}{2}} \exp \{i\mathbf{q} \cdot \mathbf{r}\}, \quad t_\mathbf{q} = q^2/2\mu,$$

where $\mu$ is the reduced mass. The $\mathcal{A}(\mathbf{Q} - \mathbf{P})$ and $a(q, p)$ in eq. (1) represent suitable wave packet amplitudes corresponding to the motion of the centre of mass and the relative motion, respectively. Moreover, we consider that $a(q, p)$ is the amplitude corresponding to the $l$-th wave state and has a strong peak at the energy $\epsilon_\mathbf{q}$. Then it may be written as

$$(3) \quad a(q, p) = \sum_{m=1}^{\infty} Y_{lm}(\mathbf{\hat{r}}) Y_{lm}(\mathbf{\hat{q}}) a_l(t_\mathbf{q} - t_\mu),$$

where $Y_{lm}$ denotes spherical harmonics, and the symbol $\hat{\cdot}$ on a vector denotes the unit vector of it.

The wave function $\Psi(t)$ is the exact solution of the Schrödinger equation $i\partial \Psi(t)/\partial t = H\Psi(t)$ as

$$(4) \quad \Psi(t) = \int d\mathbf{Q} \, \mathcal{A}(\mathbf{Q} - \mathbf{P})(2\pi)^{-\frac{3}{2}} \exp \{i(Q \cdot R - E_0 t)\} \int \! d\mathbf{q} \, a(q, p) \psi_\mathbf{q}(r) \exp \{-i\epsilon_\mathbf{q} t\},$$

where $\psi_\mathbf{q}(r)$ satisfies

$$(5) \quad -(1/2\mu) \nabla_r^2 + V(r) \psi_\mathbf{q}(r) = \epsilon_\mathbf{q} \psi_\mathbf{q}(r).$$


(11) Here $\alpha \cdot \alpha$ denotes the range of potential: case I: deep square-well potential, $\delta_\alpha = (\pi/2) - qa$, (see ref. (11)); case II: Hultén type potential, $\delta_\alpha = (\pi/2) - (2r)qa$, ($r = 1, 2, 3, ...$) (see S. Weinberg: Phys. Rev., 131, 410 (1963)).
