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Summary. — The massive photon-torsion interaction is used to build the generalization of electromagnetic fields. It is shown in the static case in an isotropic dielectric the electric field can be written in terms of the torsion vector. In the case of propagation of electromagnetic fields and radiation gauge $A^0 = 0$, the electric and magnetic fields are expressed in terms of the torsion four-vector $Q^\mu$.

PACS 03.50.De – Maxwell theory: general mathematical aspects.

1. – Introduction.

Recently I have suggested [1, 2] that the construction of a massive photon-torsion interaction through a Proca-Cartan-type Lagrangian, could lead to some interesting features as the building of a superconductivity [3] model based on torsion, where in London equations the penetration depth in superconductor would be proportional to the divergence of the torsion vector field in three dimensions $Q$. More recently Cinelli de Oliveira and myself [4] have shown that the polarization vector $P$ could be written as

\begin{equation}
P = \frac{3\lambda e}{4\pi k} \frac{\exp[-\mu r]}{r} Q,
\end{equation}

where $\lambda$ is the photon-torsion interaction, recently shown by de Sabbata, Sivaram and myself [5] to have an upper limit of $\lambda \sim 10^{-12}$. Formula (1) is similar to a formula deduced by de Sabbata and Gasperini [6] some years ago which gives the magnetic field $H$ in terms of the torsion vector as

\begin{equation}
H = -\frac{4\pi e}{3\pi} \frac{r}{Q}.
\end{equation}

It is important to note that de Sabbata and Gasperini have considered a gauge-invariant Maxwell electrodynamics with torsion and apart from their electrodynamics, the electric fields would not depend on torsion at least in the case of...
spinning charged particles at rest. In this paper I show that by considering a massive electrodynamics with torsion\[1, 2\] one is able to deduce an electric field which is proportional to the torsion vector field and, besides, that the magnetic field in the radiation gauge can be expressed in such way that it is orthogonal to the electric field like in the propagation of electromagnetic waves. Since we are only interested in the torsion effects and not in metric effects, we consider just the Minkowsky background plus torsion vector fields. It is still important to stress that here we are considering a dynamical theory of torsion where torsion does propagate and not a case of spin-spin contact interaction as in the Einstein-Cartan case\[6\].

2. - The Proca-Cartan field equations.

Let us consider the field equations of a massive electrodynamics with torsion

\begin{align*}
\nabla \cdot D &= 4\pi\rho', \\
\nabla \cdot B &= 0, \\
\nabla \times E &= -\frac{1}{c}\frac{\partial B}{\partial t}, \\
\nabla \times H &= J - \frac{3\lambda}{k}(\nabla \cdot Q)A + \frac{1}{c}\frac{\partial D}{\partial t},
\end{align*}

where, as shown in ref.\[4\], the electric displacement \( D \) written as \( D = E + 4\pi P \), \( P \) being the polarization vector given by formula (1) and in eq. (3a) the effective charge density \( \rho' \) could be written in terms of the torsion vector \( Q \) as

\begin{equation}
\rho' = \rho - \frac{3\lambda e}{k}Q \cdot \nabla\left(\exp\left[-\frac{\mu r}{r}\right]\right),
\end{equation}

e being the electronic charge and \( \mu \equiv m_\gamma c/h \) the parameter connected with the photon mass \( m_\gamma \).

For the system (3) to be complete we should add the Proca equation itself

\begin{equation}
(\Box + \mu^2)A^\mu = 0,
\end{equation}

where \( A^\mu (\mu = 0, 1, 2, 3, ) \) is the electromagnetic four-potential.

In the special case of spinning charges at rest the following simplifications occur:

\begin{equation}
J = 0, \quad \frac{\partial D}{\partial t} = 0, \quad \frac{\partial B}{\partial t} = 0
\end{equation}

since there is no current and the electric and magnetic fields are not time-dependent.

Substitution of eqs. (6) into (3) yields

\begin{align*}
\nabla \cdot D &= 4\pi\rho', \\
\nabla \cdot B &= 0, \\
\nabla \times H &= -\frac{3\lambda}{k}(\nabla \cdot Q)A \equiv J_L,
\end{align*}