Embedding Methods for $\varphi^4$-Interaction.

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... one feels that an additional difficulty is caused by the lack of a sufficiently general and flexible formulation of the problem.


Summary. — The idea of embedding a given theory in a class of similar theories is applied to quantum field theory in the case of $\varphi^4$-interaction to derive different equations for the generating functional. The number of possible embeddings has been restricted by demanding that for the defined projections of the generating functional a closed system of equations be obtained.

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1. — Introduction.

An original theory described by a certain set of constants such as masses, coupling constants, values of dimensions and spins may be embedded into a defined set of similar theories, if some of the constants are treated as parameters varying in certain domains. This kind of embedding could be called the constant variation method.

Another method of embedding the original theory, used in this paper, consists in introducing an extra parameter(s) to the theory. An example of such an embedding is stochastic quantization (1), or the idea of extra dimension used in (2).

In the present paper we consider the quantum field theory (QFT) of one scalar field governed by $\varphi^4$-interaction. We assume that the original theory of $\varphi^4$-interaction is described by the corresponding action integral $S[\varphi]$ which we embed in a one-parameter set of action integrals $S[\varphi; t]$ which are polynomials in $\varphi$ of the same order as the original action integral describing $\varphi^4$-interaction. By means of such a set of the action integrals we build up a one-parameter set of corresponding generating functionals whose elements are connected by a Cauchy-type equation (CTE). Different embeddings are described by different sets or by different CTE.

It turns out that by dealing with generating functionals generating symmetrical $n$-p.f.s, like Green's functions it is possible to carry out another kind of embedding, namely we can introduce quantities which are able to generate symmetrical as well as nonsymmetrical $n$-p.f.s., see appendix A. In other words we look for general solutions to equations like the Schwinger equations in a space $F$ made up of nonsymmetrical $n$-p.f.s, in which symmetrical $n$-p.f.s, form a subspace. In space $F$ it is easy to construct right inverse operators to the whole class of interacting operators describing equations for $n$-p.f.s. by means of which general solutions in $F$ can be constructed, see (7). In the present paper, in space $F$, we construct right inverse operators to interacting operators describing $\varphi^4$-interaction in the cases of different embeddings to derive various CTE for $n$-p.f.s. Subsequently, arbitrary embeddings are restricted by the demand that the CTE obtained are closed for a defined set of projections of the generating functional $V[\alpha; t]$.

**Contents of work.** In sect. 2 considered embeddings of the theory of $\varphi^4$-interaction are described.

In sect. 3 and 4 equations used for the generating functional $V$ are derived and described in the vector notation. Complementary to these sections are appendices A and B in which the notion of the linear functional is used and its necessity for the equivalent description of certain equations is explained.

In sect. 5 and appendix C right invertible embeddings are described and different CTE are derived.

In sect. 6 strong and weak closure conditions are postulated and an appropriate equation for the generating functional $V$ in the case of one closure condition is derived. In appendix D we give the symmetrical description of that equation in the case of strong closure condition (D.3). A weak closure condition (6.7) is considered in appendix E and solved for the particular set of embeddings. The obtained results are used in appendix F to derive partial differential equations for a certain set of projections of the generating functional $V$.

**The main idea** running through the paper is an attempt to derive closed equations with initial conditions which can be more easily constructed than

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