Some Exact Inequalities Involving $S$ and $P$ Waves and Their Derivatives for $\pi K$ Scattering (*)

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Summary. — Inequalities involving the $S$ and $P \pi K$ partial-wave amplitudes and their derivatives are obtained via the methods used in recent works on pion-pion scattering.

1. - Introduction.

The theoretical requirements imposed by analyticity (and thus crossing symmetry) and unitarity have been widely studied recently for the fundamental example of $\pi - \pi$ scattering (1). Among the different works, those of Martin's group have led to various sets of inequalities involving the $S$ and $P$ pion-pion partial-wave amplitudes (2) and their derivatives (3). These sets of inequalities are particularly useful in the construction and the testing of theoretical models describing low-energy $\pi - \pi$ scattering amplitudes (4).

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(1) See, for example, G. Wanders: Analyticity, unitarity and crossing-symmetry constraints for pion-pion partial-wave amplitudes, Lecture note at Heidelberg-Karlsruhe International Summer Institute in Theoretical Physics, Heidelberg, July 1970.


The aim of the present work is to apply the methods of the ref. (2,3) to \( \pi \)-K scattering. The extension is far from being trivial due to the differences of masses and isospin configuration. After a brief review of some basic formulae in the next Section, we obtain in the third Section inequalities involving \( S \) and \( P \) \( \pi \)-K partial-wave amplitudes. In Sect. 4 we derive other inequalities involving the derivatives of the partial-wave amplitudes. All results are given in Sect. 5 with a short discussion in it.

The progress made recently in the construction of low-energy models of \( \pi \)-\( \pi \) scattering has let us hope that our work is not only an academical training but will also be a useful tool in the study of low-energy \( \pi \)-K scattering, joining to the already known sum rules implied by crossing symmetry on partial-wave amplitudes (5).

2. - Kinematics, isospin and amplitudes of \( \pi \)-K scattering.

We just recall here well-known results which will be useful for the sake of completeness and notations.

2'1. Kinematics and isospin. - The three channels used in the dispersion relation treatment are labelled below with the corresponding energies:

\[
\begin{align*}
\pi K &\rightarrow \pi K \quad (s), \\
\pi K &\rightarrow \pi K \quad (u), \\
\pi \pi &\rightarrow K\bar{K} \quad (t),
\end{align*}
\]

\( s + t + u = \Sigma = 2(m^2 + \mu^2); \quad m (\mu) \) is the \( K (\pi) \) mass.

The invariant \( K \pi \) amplitudes \( A^\pm(s, t, u) \) are defined from the total \( T \)-amplitude by, \( \alpha \) and \( \beta \) being the isospin indices of the incoming and outgoing pions,

\[
\begin{align*}
T_{\alpha\beta} &= \delta_{\alpha\beta} A^+(s, t, u) + \frac{i}{2} [\slashed{\mathcal{F}}_\alpha, \slashed{\mathcal{F}}_\beta] A^-(s, t, u).
\end{align*}
\]

The invariance under the exchange of two pions gives

\[
A^\pm(s, t, u) = \pm A^\pm(u, t, s).
\]

Normalization is such that the differential cross-section is

\[
\frac{d\sigma}{d\Omega} = \frac{1}{s} |T(s, t)|^2.
\]