Modified Hypothesis of Partially Conserved Axial Vector Current, the Pion-Decay Constant $f_\pi^a$ and the Ratio $f_\pi^a/f_\pi^a$ (*)(**)  

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Summary. — The hypothesis of partial conservation of weak axial vector current $\partial_\mu a_\mu^a = c_\pi(m_\pi^2 q^\pi + \mathcal{R}(x))$ is modified to the form $\partial_\mu a_\mu^a = c_\pi(m_\pi^2 q^\pi + \mathcal{R}(x))$, where $\mathcal{R}(x)$ is a pseudoscalar current operator. Invoking this modified hypothesis in the theoretical calculation of the pion-decay constant $f_\pi^a$ and the ratio of kaon- to pion-decay constants $|f_K^a/f_\pi^a|$ it is found that $f_\pi^a = 0.139m_\pi$ and $|f_K^a/f_\pi^a| = 1.28$, which are in excellent agreement with the corresponding experimental value $0.14m_\pi$ for $f_\pi^a$ and the Cabibbo-angle value $1.28$ for $|f_K^a/f_\pi^a|$.

1. — Introduction.

In recent years, the hypothesis of partially conserved axial vector current (PCAC) has been greatly used to study the weak interaction of strongly interacting particles. The essential prediction of this hypothesis is the famous Goldberger-Treiman (G.T.) relation (1)

\[
(f_\pi^a = \frac{\sqrt{2} m_\pi F_\pi^a(0)}{g_s},
\]

which gives a remarkable connection between pionic phenomena and the axial vector renormalization constant $F_\pi^a(0)$. Here, $f_\pi^a$ is the pion-decay constant, $m_\pi$ is the mass of the proton, and $g_s$ is the strong pion-nucleon coupling constant.

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If we put in eq. (1.1), the recent experimental values (2) of \( m_\pi, \ F_{A}^{\pi}(0) \) and \( g_s \), then we get

\[
(1.2) \quad f_A^\pi = 0.128 \ m_\pi ,
\]

which has to be compared to the experimental value (3) of \( f_A^\pi = 0.14 \ m_\pi \).

Extension of the PCAC hypothesis to the strangeness-violating decays yields a similar connection between the kaon-decay constant \( f_A^K \) and the axial vector renormalization constant \( F_{A}^{\pi}(0) \). This is expressed as

\[
(1.3) \quad f_A^K = \frac{(m_\pi + m_\Lambda) F_{A}^{\pi}(0)}{\sqrt{2} g_s^{K\Lambda}} = \frac{(m_\pi + m_\Lambda) F_{A}^{\pi}(0)(1 + 2\alpha)}{\sqrt{2} g_s(1 + 2\beta)} ,
\]

where the last equality follows from the \( SU_3 \) limits of the weak and strong kaon-nucleon coupling constants \( F_{A}^{K\Lambda} \) and \( g_s^{K\Lambda} \) respectively. Here \( \alpha \) and \( \beta \) are defined in such a way that \( (1 - \alpha)/\alpha \) and \( (1 - \beta)/\beta \) respectively denote the \( (D/F) \) ratios for the weak and strong hadronic currents. From eqs. (1.1) and (1.3) we get the ratio

\[
(1.4) \quad f_A^K/f_A^\pi = \frac{(m_\pi + m_\Lambda)(1 + 2\alpha)}{2m_\pi (1 + 2\beta)} .
\]

In the limit of good \( SU_3 \), this ratio is unity. However if we use in eq. (1.4) the physical masses for \( m_\pi \) and \( m_\Lambda \) and equate \( \alpha = \beta \), we obtain

\[
(1.5) \quad |f_A^K/f_A^\pi| = 1.09 ,
\]

which is low compared to the Cabibbo angle value (4) of 1.28 (5). From the above analysis it seems that, although the hypothesis of partial conservation of axial vector current can be applied with certain reliability to \( \Delta S = 0 \) decays, perhaps it cannot be applied with that much reliability to \( \Delta S = 1 \) decays. Therefore we propose a modification of the hypothesis, so that the latter will be equally good both for strangeness-conserving and strangeness-non-conserving decays, and yields better values for \( f_A^\pi \) and \( |f_A^K/f_A^\pi| \).


(4) The number 1.28 for \( f_A^K/f_A^\pi \) quoted from ref. (4) is obtained by assuming that the vector currents are not renormalized.