On the Formation of Envelope Solitons in Magnetized Plasmas (*)

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The linear parametric instabilities of intense electromagnetic radiation in non-magnetized plasmas are now well understood (1). Because of the importance for laser pellet fusion, ionospheric-heating experiments and heating of magnetically confined plasmas (2), parametric instabilities in magnetized plasmas are of growing interest (3-6).

Recently, the possible nonlinear saturation mechanisms for the linear parametric instabilities has received much attention. For nonmagnetized plasmas, several authors (7) have calculated the nonlinear state of the purely growing mode which is caused by self-modulation of a high-frequency wave. The equation describing the nonlinear development of the wave amplitude has been found to be the nonlinear Schrödinger equation. The latter follows from a WKB analysis taking into account the effect of group dispersion and that of quadratic nonlinearity in the susceptibility because of the low-frequency pondero-motive force. In particular, due to the pondero-motive force, locally strong fields may deplete the density producing a cavity which can trap the electric field. Thus stationary localized solutions (solitons) are possible.

Soliton solutions in magnetized plasmas have been reported so far for modulationally unstable cyclotron (8), whistler (9), upper (10) and lower hybrid (11) waves. It is the purpose of this letter to derive the explicit form of the nonlinear Schrödinger equation.

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(2) M. Porkolab: Nucl. Fusion, 12, 329 (1972).
in magnetized plasmas by a simple method which is based on results known from the
theory of linear parametric instabilities. This procedure allows us to study a new type
of upper hybrid solutions which, to our knowledge, has not been reported previously.

An electrostatic high-frequency wave with finite amplitude $E$, frequency $\omega$ and
wave vector $k$ may obey a nonlinear dispersion relation

$$ D(\omega, k; |E|^2) = 0 . $$

Then, it has been shown (12,13) that the nonlinear state of the wave amplitude is described
by the nonlinear Schrödinger equation

$$ i \frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} \frac{\partial^2 E}{\partial k^2} - \frac{\partial \omega}{\partial |E|^2} |E|^2 E = 0 , $$

where

$$ \xi = x - v_g t , $$

and $v_g$ is the linear group velocity of the wave packet. For reasons of simplicity we
have assumed a one-dimensional description.

For a magnetized plasma one can determine the nonlinear frequency shift $\partial \omega/\partial |E|^2$
from the dispersion relation known in linear parametric instability problems (2,14), i.e.

$$ D(\omega, k; |E|^2) = \varepsilon(\Omega) + (\mu^2/4) \chi_i(\Omega)[1 + \chi_e(\Omega)](\varepsilon^{-1}(\Omega - \omega) + \varepsilon^{-1}(\Omega + \omega)) = 0 , $$

where

$$ \varepsilon(\Omega \pm l\omega) = 1 + \chi_i(\Omega \pm l\omega) + \chi_e(\Omega \pm l\omega) $$

is the linear dielectric function, $l = 0, \pm 1$, and $\chi_e,i$ is the linear electron or ion suscep-
tibility in a magnetized plasma. The coupling coefficient $\mu$ is given by

$$ \mu = \frac{e}{m} \left\{ (E_x K_x / \omega^2 + (E_x K_x + E_y K_y) / (\omega^2 - \omega_{ge}^2))^2 + (E_x K_y - E_y K_x)^2 \omega_{ge}^2 / (\omega^2 - \omega_{ge}^2) \omega_{ge}^2 \right\}^{1/2} , $$

where $\omega_{ge}$ is the electron gyrofrequency, $e$ is the absolute value of the charge of an electron,
and $m$ is the electron mass. The external magnetic field has been assumed to be in the
$z$-direction, and $\Omega$ and $K$ are the frequency and the wave number of the modulation,
respectively.

We now calculate the nonlinear frequency shift for the special case of an upper
hybrid mode near the resonance frequency, propagating approximately perpendicular
to the external magnetic field. Then the frequency is given by (15)

$$ \omega(k) = \omega_H + v_g A \omega_{ge}^2 k^2 / 2 \omega_{ge}^3 , $$


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\(^{(15)}\) A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko and K. N. Stepanov: Plasma