Structure of Leptons and Mesons.

G. Rosen (*)
Institute for Theoretical Physics - Stockholm

(riccavuto il 4 Agosto 1960)

This paper reports a quantum theory for the structure of leptons and mesons. The quantum theory is presented in axiomatic fashion without recourse to a classical Hamiltonian theory.

The state vector of a lepton or meson is written

\[ \psi = \psi_{\text{ext}} \psi_{\text{int}}, \]

where \( \psi_{\text{ext}} \) and \( \psi_{\text{int}} \) are the external and internal states of the particle. The external state \( \psi_{\text{ext}} = \psi_{\text{ext}}(x) \) can be represented by a set of complex-valued functions of the space-time co-ordinates \( x = (x_1, x_2, x_3, x_4) \); \( \psi_{\text{ext}} \) satisfies a well-known wave equation for a lepton or meson of a given mass and spin. Similarly, the internal state \( \psi_{\text{int}} = \psi_{\text{int}}(\xi) \) can be represented by a set of complex-valued functions of four real (dimensionless) internal co-ordinates \( \xi = (\xi_1, \xi_2, \xi_3, \xi_4) \). The space-time co-ordinates \( x \) and the internal co-ordinates \( \xi \) are mutually independent variables. Not subject to space-time transformations and therefore Lorentz invariant scalars, the momenta conjugate to the \( \xi \)'s define the structural observables of the particle:

\[ N_L = -i \frac{\partial}{\partial \xi_1}, \]

\[ N_s = -i \frac{\partial}{\partial \xi_2}, \]

\[ s = -\frac{1}{2} \hbar \left( \frac{\partial}{\partial \xi_1} \right)^2, \]

\[ q = -ie \left( \frac{\partial}{\partial \xi_2} + \frac{\partial}{\partial \xi_3} \right), \]

\[ m = m_n \left( \frac{\partial}{\partial \xi_4} \right)^4. \]

(*) National Science Foundation Postdoctoral Fellow.
A physically admissible \( \psi_{1,2} \) which is also a simultaneous eigenstate of the observables defined by eqs. (2) characterizes a single lepton or meson. Physically admissible internal states are determined by a subsidiary condition and a wave equation, relationships formulated in the following paragraphs.

1. Subsidiary condition.

The subsidiary condition involves the internal co-ordinates \( \xi_1, \xi_2, \xi_3 \) symmetrically, and \( \xi_4 \) makes no appearance

\[
(A' + 1)\psi_{\text{int}} = 0 ,
\]

where

\[
A' = WV^{-1}U' .
\]

\[
U = \sum_{a=1}^{3} \frac{\partial^2}{\partial \xi_a^2} ,
\]

\[
V = \sum_{a=1}^{3} \frac{\partial}{\partial \xi_a} ,
\]

\[
W = \sum_{a=1}^{3} \frac{\partial}{\partial \xi_a} .
\]

In eq. (4) \( V^{-1} \) denotes the formal inverse of the differential operator defined by eq. (5b).

There is a similarity between the subsidiary condition (3) and a Helmholtz equation, obtained if \( A' \) in eq. (3) were replaced by \( \Lambda = \sum_{a=1}^{3} \frac{\partial^2}{\partial \xi_a^2} \). Both \( A' \) and \( \Lambda \) have the same homogeneity with respect to \( (\xi_1, \xi_2, \xi_3) \); this fact is expressed by the commutation relations

\[
[W, A'] = -2A' , \quad [W, \Lambda] = -2\Lambda ,
\]

in which \( W \) is defined by eq. (5c). Moreover, for a function \( f \) which is analytic in \( \xi_1, \xi_2, \xi_3 \) in the neighborhood of the origin, it follows that

\[
A'f = WV^{-1}U \left( f (\xi_1, \xi_2, \xi_3) + \sum_{a=1}^{3} \left( \frac{\partial f}{\partial \xi_a} \right) \frac{\partial}{\partial \xi_a} \xi_a + \frac{1}{2} \sum_{a=1}^{3} \sum_{b=1}^{3} \left( \frac{\partial^2 f}{\partial \xi_a \partial \xi_b} \right) \xi_a \xi_b + \ldots \right) =
\]

\[
= WV^{-1} \left( \sum_{a=1}^{3} \left( \frac{\partial^2 f}{\partial \xi_a^2} \right) \xi_a + \ldots \right) = W \left( \sum_{a=1}^{3} \left( \frac{\partial^2 f}{\partial \xi_a^2} \right) \ln \xi_a + \ldots \right) = \sum_{a=1}^{3} \left( \frac{\partial^2 f}{\partial \xi_a^2} \right) + \ldots .
\]

Thus, \( A' \) acts like \( \Lambda \) in the neighborhood of the origin,

\[
(\Lambda f)_{\xi = 0} = (Af)_{\xi = 0} .
\]

The subsidiary condition (3) assumes a more tractable form with the elimination of the integration operator \( V^{-1} \). Applying the operator \( V \) to eq. (3) and using the