Scattering of Electrons by a Cylindrical Potential Well in a Powerful Low-Frequency Radiation Field.

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Summary. — We consider the scattering of electrons by a cylindrical potential well in a powerful low-frequency radiation field. As a special case we also discuss the scattering by a ,,hard)) cylinder. The electrons are described nonrelativistically, and the laser field is treated in the dipole approximation. This puts certain restrictions on the intensity of the laser. By choosing the laser field to be circularly polarized and to propagate in the direction of the axis of the cylinder (taken as the z-axis), our two-dimensional scattering process in the (x,y)-plane has a large amount of symmetry. If the energy of the laser quanta is much less than the energy of the scattered electrons, the above problem can be solved exactly, and explicit analytic expressions can be derived, in particular for the differential and total scattering lengths \( dL_n \) and \( L_n \) respectively, describing the scattering of electrons while they are absorbing or emitting \( n \) quanta of the laser pulse. Our present paper is a straightforward generalization of our earlier work on laser-assisted scattering by a one-dimensional square well potential (J. Opt. Soc. Am., 7, 537 (1990)) and it yields valuable information on the angular distributions of scattered electrons in a powerful radiation field.

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1. – Introduction.

Scattering of electrons by potentials or atoms in the presence of a laser field has been investigated by many authors in various contexts. Several reviews have appeared on this subject and we just quote a few of them in which different aspects of the problem are considered [1-4]. Moreover, we mention the books by Faisal [5] and by Mittleman [6]. Since the general problem of laser-assisted electron scattering by an atomic system is rather complicated to handle, various different approximation schemes have been considered and a number of model calculations have been performed [7-14], giving insight into important features of electron scattering in a
powerful laser field. In all the very recent calculations a one-dimensional model has been treated [11-14].

A few years ago we tried to solve the problem of electron scattering by an impenetrable sphere in a laser wave [10]. Although we succeeded in writing down spherical Gordon-Volkov states and found a convenient Floquet-ansatz for the ingoing and scattered electron waves in the laser field, the solution of the matching equations at the surface of the sphere caused great difficulties mainly due to the lack of symmetry of the problem. Hence our solution was insatisfactory. In the meantime, we solved exactly and with similar techniques the one-dimensional problem of laser-assisted electron scattering by a square well potential in the low-frequency limit [12].

In the present paper we would like to show that a similar exact solution can be found for the two-dimensional problem of electron scattering by a "hard" or "soft" cylinder in a laser field if the scattering geometry is conveniently chosen to obtain a maximum of symmetry. This is achieved by taking the laser field to be circularly polarized and to propagate in the direction of the cylinder axis, which is chosen as the z-axis of our coordinate system. By this choice our problem has rotational symmetry which enormously facilitates the calculations. Scattering will then take place in planes perpendicular to the axis of the cylinder, if the space-dependence of the laser field is neglected. We shall treat simultaneously the laser-assisted scattering by an infinitely long impenetrable cylindrical potential well and by a cylindrical potential well of finite depth. The first case, of course, is simpler than the second. It is, moreover, interesting to observe that it is easier to treat the matching equations on the single boundary of the cylindrical well than on the two boundaries of the one-dimensional well, yielding in the former case only two equations to be solved instead of four in the latter case.

In sect. 2 we shall first derive cylindrical Gordon-Volkov states in analogy with the spherical Gordon-Volkov states of our earlier work [10]. The next section, 3, will be devoted to the solution of the cylindrical scattering problem in the absence of the laser field. Here we shall first present a time-independent cylindrical partial wave analysis and shall then treat the same problem by appropriately adapted time-dependent scattering theory, which turns out to be more appropriate for the solution of the same problem in a laser field and which will be taken up in sect. 4. In sect. 5 we shall summarize our main results and compare them with previous investigations.

2. – Cylindrical Gordon-Volkov states.

The Schrödinger equation of an electron of charge $-e$ and of mass $M$, interacting with a right-circularly polarized monochromatic electromagnetic wave (henceforth our laser field) reads

$$\frac{1}{2M}(-\vec{p} + (e/c)\vec{A})^2\psi = i\hbar\partial_t\psi,$$

where

$$A(t) = (cF/\omega)(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$$

is the vector potential describing the laser field of frequency $\omega$ and of electric field strength $F$. This field is assumed to propagate along the positive z-direction and