Theories with Limited Acceleration: Free Fields.

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Summary. — We continue the study of a geometric scheme which takes into account the limitations to the acceleration of material bodies which follow from quantum-gravitational effects. After an analysis of the geometric structure and of its symmetry group $GL(4, \mathbb{R})$, we introduce some $GL(4, \mathbb{R})$-invariant linear field equations in the 10-dimensional space $\mathcal{J}$ of the local reference frames. We discuss both the classical and the quantized version of these field theories in a flat space $\mathcal{J}$ and on the manifold of the symplectic group $Sp(4, \mathbb{R})$. In the physically interesting case in which $\mathcal{J}$ is the bundle of the orthogonal frames of Minkowski space-time, these field theories present some inconsistencies, which are analysed in detail. They seem to be unavoidable in a theory which disregards quantum-gravitational effects, but is based on a geometric scheme which contains some consequences of these effects. We suggest that the terms which generate the inconsistencies can be compensated by quantum correlations if the gravitational coupling constant has a value determined by the geometry.

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1. – Introduction.

An important feature of the quantum-relativistic theories of gravitation is the possibility of defining natural units for all the physical quantities in terms of the velocity of light $c$, the Planck constant $\hbar$ and the Newton constant $G$. In particular one can define a fundamental length $l_p$ (Planck's length) and a fundamental acceleration $a_p$ given by

$$l_p = \sqrt{\frac{\hbar G}{c^5}}, \quad a_p = \sqrt{\frac{c^5}{\hbar G}}.$$
In the following, we use the convention $h = c = 1$. It has been suggested that there is a lower limit of the order of $\ell_p$ to the accuracy of any measurement of length and that there is an upper limit of the order of $a_p$ to the acceleration of material bodies.

The existence of a limit to the accuracy of time and position measurements has been proposed a long time ago; a clear derivation of this limit from the principles of general relativity and quantum mechanics is given in ref. The existence of a maximal acceleration has been suggested by several authors. Its first justification as a quantum-gravitational effect has been given in ref. by combining an upper limit to the temperature with the known results on the temperature measured by an accelerated observer. Another argument is given in ref. One can also remember that an accelerated reference frame with acceleration $a$ can be defined only at distances smaller than $a^{-1}$; then the maximal acceleration can be derived from the existence of a minimal measurable distance. All these arguments are based on the ideas of quantum theory and of general relativity; since a consistent theory of quantum gravity is not yet available, they have necessarily a qualitative nature.

Einstein's theory of relativity owes its elegance and coherence to the fact that its main feature, the existence of an upper limit to the velocity of material bodies, is introduced from the beginning in the underlying geometry and in the symmetry group. It is conceivable that the difficulties found in the formulation of a quantum theory of gravitation are due to the fact that the further limitations indicated above are not contained in the geometric structure and in the symmetry properties which are used as a starting point for the construction of the theory.

A geometry which takes into account limitations to the accuracy of length and time measurements cannot be based on a space-time differentiable manifold and one may have to give up the methods of differential geometry. A less drastic approach is to describe the physical phenomena by means of a manifold.

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(1) W. Heisenberg: Z. Physik, 110, 251 (1938).