Standard Model Gauge Group and Realistic Fermions from the Most Symmetric Coset $M^{klm}$

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Summary. — A brief survey on Kaluza-Klein theories is given. It is emphasized that for a maximally supersymmetric universe, the only way to do Kaluza-Klein theory is through supergravity. Bearing this in mind, a seven-dimensional internal compact manifold, $M^{klm}$, is considered. It is described as being the most general type of coset spaces containing the standard model groups $SU(3) \otimes SU(2) \otimes U(1)$. $M^{klm}$ is obtained as a compactification of $D = 11$ supergravity. It is shown that contrary to the manifolds $M^{pqrs}$, the topology of the spaces $M^{klm}$ is not sensitive to the integers, $k$, $l$, $m$ but rather to the 6 parameters that parametrize the subalgebra $SU(2) \otimes U(1)^{'} \otimes U(1)^{''}$ taken to be the stability group of $M^{klm}$. This gives the spaces $M^{klm}$ a much richer topology and thus a much wider class of solutions of $D = 11$ theory. Also, the most general expressions for the "geometrical" coupling constant are obtained. A consistency condition with the low-energy standard model, with and without supersymmetry, is established. A harmonic expansion analysis is made, it is shown that $M^{klm}$ manifold possesses a spinor structure for a given class of parameters. It is shown that for two classes of parameters $L_{0z}^{klm}$ and $L_{0y}^{klm}$ the manifold $M^{klm}$ becomes the 8-dimensional coset space $L^{klm}$ for which classes it is seen to be consistent with the $SU(3) \otimes SU(2) \otimes U(1)$ quark and lepton quantum numbers. The $SU(3) \otimes SU(2) \otimes U(1)$ invariant solutions of $D = 11$ supergravity are studied. It is shown that all solutions break supersymmetry except for two mirror solutions for which $N = 2$ supersymmetry is preserved. The 8-dimensional coset manifold $L^{klm}$ is then considered; it is obtained as a minimal increase on the manifold $M^{klm}$.

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1. - Introduction.

The quest for a unified field theory of the fundamental interactions, gravity, electromagnetism and the weak and strong nuclear forces, is not a new aim. Already in the 1920's Kaluza and Klein suggested a unification of the gravitational and
electromagnetic interactions which, at the time, were the only forces to be well understood. They showed within the context of a five-dimensional extension of Einstein's theory of general relativity how both electromagnetism and gravity could be treated on a similar footing in that both are described as parts of the five-dimensional metric. Electromagnetic gauge transformations are interpreted as coordinate transformations in the extra dimension which preserve the Kaluza-Klein form of the metric.

It was later demonstrated by Salam and Weinberg how the weak and electromagnetic interactions could be unified in a non-Abelian gauge group \( SU(2) \otimes U(1) \). More ambitious theories try to include the strong interactions in a grand unified theory such as the \( SU(5) \) model of Georgi and Glashow. Such theories exhibit many arbitrary features and contain many free parameters. Also, there is no obvious way to incorporate gravity. Considering the non-Abelian generalisation of Kaluza-Klein theories, the logical step is to include the strong and weak interactions.

Intuitively we require the extra dimensions to be the group manifold of the internal gauge group \( G \). If we want \( G \) to contain \( SU(3) \otimes SU(2) \otimes U(1) \), then the group manifold must be at least twelve-dimensional. However, with the introduction of supersymmetry, it is believed that in four dimensions there are no consistent massless interacting-field theories with spins greater than two. Using this spin restriction it was observed that the structure of the supersymmetry algebras placed an upper limit of \( D = 11 \) on the dimensions of space-time in which a consistent supersymmetric field theory can be formulated. In this case, the Lorentz group is \( SO(1, 10) \) and the Dirac spinor index runs from 1 to 32. This corresponds in \( d = 4 \) to eight four-component spinors. There are, therefore, \( N = 8 \) supersymmetry generators, each one changing the helicity of physical states by half a unit. In \( d = 4 \) the helicities take the values: \(-2, 3/2, -1, -1/2, 0, 1/2, 1, 3/2, 2\). Therefore, spin \( \leq 2 \) corresponds to \( N \leq 8 \) and, hence, to \( D \leq 11 \). If nature is maximally supersymmetric, then \( D = 11 \) dimensions are to be taken seriously.

The corresponding \( D = 11 \) supergravity Lagrangian describes the interaction of gravity \( e_A^M \) (elfbein) which in \( D = 11 \) has 44 degrees of freedom, with a gravitino \( Y_M \) which has 128. Since supergravity requires equal numbers of bosons and fermions we must, therefore, have another bosonic field with 84 degrees of freedom. This corresponds to a three-index antisymmetric tensor gauge field \( A_{MNP} \). The theory consists of \( D = 11 \) general covariance, local \( N = 1 \) supersymmetry, local \( SO(1, 10) \) Lorentz invariance and Abelian gauge invariance of \( A_{MNP} \) and a discrete symmetry which performs an odd number of space or time reflections such that \( A_{MNP} \rightarrow -A_{MNP} \). Since supersymmetry forbids the presence of any other fields, the resulting equations of motion are unique. The \( D = 11 \) theory also fulfils the Kaluza-Klein requirement in that its symmetries are space-time symmetries. Another positive feature is that it does not present the arbitrariness of the grand unified theories. It has only one parameter, the gravitational coupling constant. The next logical step is to construct the extended supergravity Lagrangian in \( d = 4 \) corresponding to the \( N = 8 \) theory mentioned above. To this end, the technique of dimensional reduction was used. This requires writing down the \( D = 11 \) Lagrangian and then taking all fields to be independent of the extra seven dimensions. The internal space then corresponds to the seven-torus \( T^7 \). By performing duality and other field transformations the internal space has an \( E_7 \otimes SU(8) \) symmetric form which was gauged giving \( SO(8) \otimes local \ SU(8) \) symmetry. At this point the extra dimensions