Bounds on Multichannel Scattering Parameters.

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Summary. — It is shown that definite bounds on a reaction matrix could be obtained by using the technique of asymptotic separation recently developed for the description of rearrangement collisions and processes above the threshold of the division of the system into three fragments.

1. — Introduction.

The general scheme of the so-called unified theory of nuclear reactions is the following. The wave function $\Psi$ of the considered many-body system is expanded in some proper set (1) of well-known basic functions $\Phi_\beta$

$$\Psi = \sum_\beta F_\beta \Phi_\beta$$

and the coefficients $F_\beta$ are determined from the Schrödinger equation by the usual procedure

$$\langle \Phi_\alpha | (H - E) \sum_\beta F_\beta \Phi_\beta \rangle = 0 ,$$

where the integration is over all the space variables of $\Phi_\alpha$.

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The central problem of this formalism is to estimate the error which is introduced by the reduction of the expansion (1) to a finite number of terms and by solution of a reduced system (2) of a finite number of equations for $F_B$.

So far, this problem has been poorly investigated (see nevertheless the article by Amirkhanov et al. (2)).

There is a well-known example in quantum mechanics when such errors can be exactly controlled. The calculations of energy levels by means of variational methods give the upper bounds for exact eigenvalues. And although these bounds are informative only if they are violated, it is possible in this case to draw rigorous conclusions even if the calculations are very rough.

Unfortunately, the situation for the scattering problem is much more complicated. This is because the variational principle has not in this case the character of the minimization problem, but provides only the condition $\delta f = 0$, where $f$ is the functional which gives the reaction amplitude.

Nevertheless, Spruch with collaborators (4) has shown that in multichannel formalism it is possible to obtain the bounds for the reaction matrix $K$ (for processes without rearrangement) below the threshold of division of the system into three parts. This result was generalized to the reactions with rearrangement of particles (4) using the method of separation of asymptotic components in $\Psi$ (4). Lkhagva (6) has proved the same theorem for expansion of $\Psi$ in the set of hyperharmonics.

In this paper the theorem about lower bounds on the $K$-matrix is proved for the reactions above the threshold of the division of the system into three fragments. This proof is performed in a way which is valid for reactions of the general type.

2. - The basic equations.

Let us look for the wave function $\Psi$ according to the procedure of separation of asymptotics (1-5) in the following form:

$$\Psi = \phi^{(\text{ass})} + \phi^{(\text{int})}.$$  \hspace{1cm} (3)

Here

$$\phi^{(\text{ass})} = \sum_{\alpha \beta m} F^{(3)}_{\alpha \beta m} \phi^{(2)}_{\alpha \beta m} + \sum_k F^{(3)}_k \phi^{(2)}_k.$$  


