Relativistic Action at a Distance through Singular Lagrangians with Multiplicative Potentials and its Relation to the Nonrelativistic Two-Body Problem.

A. BARDUCCI

Istituto di Fisica Teorica dell'Università - Firenze
Istituto Nazionale di Fisica Nucleare - Sezione di Firenze

L. LUSANNA and E. SORACE

Istituto Nazionale di Fisica Nucleare - Sezione di Firenze

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Summary. — Recent models of relativistic action at a distance through singular Lagrangians with multiplicative potentials, describing two-point bound states, are re-examined. They are reformulated in such a way to be well suited to the study of extended bodies; we introduce a set of vierbeins, attached to the barycentric co-ordinates, which connect the Minkowski space with an inner relative space, and we define new relative co-ordinates in it. By using the irreducible representation theory of the Poincaré group, we show that this relative space is the natural relativistic generalization of the nonrelativistic relative one. The nonrelativistic limit of these models is exhibited, by recovering the Newtonian two-body problem with central forces.

1. - Introduction.

Classical relativistic action at a distance (a.a.a.d.) has been a constant research field for many years (1-3). Fokker's (1) and Wheeler-Feynman's (2) electro-

(1) A. D. FOKKER: Zeits. Phys., 58, 386 (1929); Physica, 9, 33 (1929); 12, 145 (1932).
(3) For a complete review of the subject see The Theory of Action at a Distance in Relativistic Particle Dynamics, edited by E. H. KERNER (New York, N. Y., 1972).
dynamics is the first example of a noninstantaneous a.a.a.d. theory between two charged particles, in which one particle’s world-line is influenced only via the retarded and advanced light-cones drawn from the other world-line. Instead Van Dam and Wigner (1) proposed an instantaneous a.a.a.d. theory, in which one-particle world-line experiences the actions of all the points of the other world-line at spacelike distances; this is achieved by equating particle four-accelerations to suited four-vector "forces". These theories use a many-time (one for each particle) formulation of dynamics.

Another approach to instantaneous a.a.a.d. was based on Dirac’s Hamiltonian theory (2), i.e. on the construction of a canonical realization of the ten generators of the Poincaré group starting from canonical single-particle variables as in the noninteracting case (see ref. (3) for the various attempts after Thomas’ paper (4)). This line of research stopped due to the no-interaction theorem (5) of Currie, Jordan, Sudarshan: when the canonical co-ordinates are taken as the physical ones of point particles and are required to transform as the co-ordinates of events, the dynamics is forced to the noninteracting case (indeed manifest covariance is a strong physical assumption not implied by relativity).

To circumvent the no-interaction theorem, a Newtonian-like relativistic dynamics with an unique time was proposed (see ref. (6) and subsequent works quoted in ref. (7)); in this theory forces had to satisfy certain conditions in order to ensure world-line invariance, thus one has relativistic form invariance but not manifest covariance of the dynamics. Then one tries to arrive at a Hamiltonian description through the Lie-Königs theorem (8). Although in general a given Newtonian-like scheme can produce various inequivalent Hamiltonian systems, nevertheless one succeeds in obtaining a canonical realization of the Poincaré group; then one has to identify canonical centre-of-mass and relative co-ordinates and momenta. As is well known, the unique canonical centre-of-mass variables are the Pryce-Newton-Wigner ones (9), which are not covariant; thus the no-interaction theorem does not apply, because the centre-

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