Linear Heat Equations from Kinetic Theory.

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Summary. — Various forms of the linear heat equation which can be deduced from the kinetic theory of gases are analysed. It is shown that, if one uses a perturbative procedure based on a power series expansion in the viscosity coefficient μ (or in the mean free path λ), the resulting equations are of the «parabolic» type, which means that the propagation velocity of the thermal disturbance is always infinite. Conversely, both the equations derived by using the Cattaneo procedure and those which are directly derived from the thirteen-moment approximation introduced by Grad to solve the Boltzmann equation are all of the «hyperbolic» type with well-defined propagation velocity. The theoretical interest of the direct measurement of the propagation velocity of a thermal disturbance is also pointed out.

1. — Introduction.

Many years ago CATTANEO (1) suggested to modify the heat diffusion equation in order to overcome the difficulties arising from the parabolic character of the equation. The modification introduced was the following: to describe the inertial properties of the thermal flow, Fourier's law of heat conduction

\[ q = - k \text{grad } \theta \]

has to be changed by adding a relaxation term proportional to \( \dot{q} \). Indeed a relaxation term was first considered by MAXWELL (2), but he disregarded this

term because its influence on stationary conditions disappears rapidly with time.

By taking into account the relaxation term, the heat conduction law becomes

\[ q = -k \text{grad} \theta - \tau \frac{\partial q}{\partial t} \tag{2.1} \]

and, combining eq. (2.1) with the first law of thermodynamics, one obtains the following heat equation:

\[ \frac{\partial \theta}{\partial t} = D \Delta \theta - \tau \frac{\partial^2 \theta}{\partial t^2} \tag{3.1} \]

where \( D = k/\rho c_v \) is the so-called thermometric conductivity and \( \Delta \) is the usual Laplace operator. In contrast to the classical heat diffusion equation, eq. (3.1) is of hyperbolic type and is known in the literature as « the telegraph equation ». Such a circumstance overcomes the unpleasant feature of the usual heat equation according to which every thermal disturbance of the medium spreads instantaneously to each point of the space.

Ten years later Vernotte (3) and Cattaneo (4) re-examined the problem of the heat equation and presented again eq. (3.1). Successively many authors took up Cattaneo’s suggestion inspecting its consequences from different points of view (5-13). Equation (3.1) is nowadays admitted as a « good » alternative to the classical diffusion heat equation and Lukov, in his textbook on heat diffusion theory (14), quotes the conclusions of Cattaneo’s work as an established result.

Cattaneo’s suggestion has been more recently taken up again by myself in collaboration with Morro (15), introducing a relaxation term also in the constitutive equation for the viscous stress tensor. The modified Navier-Stokes equations, which result when one introduces both relaxation terms, are now

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