A Generalization of Chiral $SW_3$ Model (*) (**).

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Summary. — Assuming the algebra of currents, PCAC for pions, and some other technical conditions, we prove that any Hamiltonian density $H(x)$, whose $SU_3$-breaking component belongs to the $(3, 3^*) \oplus (3^*, 3)$ representation of the $SW_3$-group, must automatically have the following structure:

$H(x) = H_0(x) + H_1(x) + H_2(x)$.

$H_0(x)$ is invariant under $SW_3$, $H_1(x)$ breaks $SU_3$ but is invariant under $SW_2$, while $H_2(x)$ is invariant under $SU_3$ but violates $SW_3$. Also, $H_3(x)$ will vanish in the soft-pion limit. This model contains, as special cases, the schemes of Gell-Mann, Oakes and Renner, and of Glashow and Weinberg.

1. Introduction and summary of results.

Some years ago, GELL-MANN, OAKES and RENNER (1) and independently GLASHOW and WEINBERG (2) (hereafter referred to as GMOR-GW) proposed an interesting model with the Hamiltonian density

(1.1)  

$H(x) = H_0(x) + \varepsilon_0 S^{(0)}(x) + \varepsilon_8 S^{(8)}(x)$,

where $H_0(x)$ is invariant under the chiral $SW_3$ group, and the scalar densities $S^{(x)}(x)$ together with their pseudoscalar counterparts $p^{(x)}(x)$ ($x = 0, 1, ..., 8$) belong to a $(3, 3^*) \oplus (3^*, 3)$ representation of the group. Using the local generalization of the usual equation of motion together with the algebra of cur-

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rents, they derived the following partial conservation laws:

\begin{align}
\partial_\mu V_\mu^\alpha(x) &= \epsilon_\beta f_{\alpha\beta\gamma} S_\gamma(x), \\
\partial_\mu A_\mu^a(x) &= (\epsilon_\alpha d_{a\beta\gamma} + \epsilon_\beta d_{a\beta\gamma}) P^\beta(x),
\end{align}

where the repeated index \( \beta \) implies an automatic summation over \( \beta = 0, 1, \ldots, 8 \), and \( a \) assumes values \( a = 1, \ldots, 8 \). In the above, \( V_\mu^\alpha(x) \) and \( A_\mu^a(x) \) are the usual vector and axial-vector currents whose 4th components generate the Lie algebra of the \( SW_3 \) group.

If we define \(^{(1)}\) a parameter \( a \) by

\begin{equation}
a = \epsilon_\alpha / \sqrt{2} \epsilon_0,
\end{equation}

then we find that \( H(x) \) becomes invariant under various subgroups of \( SW_3 \) such as ordinary \( SU_3 \), \( SW_3 \) and chiral \( SU_3 \) at the special points \( a = 0, -1 \) and \( 2 \), respectively. Usually, we implement the theory with the philosophy that at the \( SW_2 \) point \( a = -1 \) the pion emerges as a zero-mass Nambu-Goldstone boson. Thus the small pion mass \( m_\pi = 140 \) MeV, becomes readily understandable if \( H(x) \) is approximately \( SW_2 \)-invariant with the value of \( a \) close to \( -1 \).

Although this theory is elegant, as yet there seems to be no firm experimental evidence in favor of the model. Indeed, the original calculation \(^{(5)}\) of Kim and von Hippel which was considered to support the model has been criticized recently by several authors \(^{(4,5)}\), especially by Cheng and Dashen who obtained a value for the \( \sigma \)-term nearly four times larger. (However, several other authors \(^{(5)}\) give values for the \( \sigma \)-term which are very close to that given by Kim and von Hippel.) Thus the whole question appears still to be open. It may be worth-while to remark in this connection that the calculation by Cheng and Dashen can be still compatible with the GMOR-GW model, provided the following alternatives hold. Either \(^{(4,5)}\) the parameter \( a \) is near the \( SU_3 \) point \( a = 0 \) rather than the \( SW_2 \) point \( a = -1 \), or the scale-