A Simple Derivation of Schwinger's Sum Rule for Spin-Dependent Structure Functions (*)

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Summary. — Schwinger’s new sum rule for the spin-dependent structure functions is derived from the superconvergence principle. The Regge behaviour of the virtual photon-nucleon scattering amplitude is deduced from the Van Hove model from which a superconvergence relation is shown to follow leading to the sum rule. The saturation of the sum rule is discussed.

1. — Introduction.

Recently SCHWINGER (1) has given a source theory discussion of deep inelastic electron scattering and has derived an interesting sum rule for the spin-dependent structure functions \( W_3 \) and \( W_4 \). This sum rule can be written as

\[
\int_0^\infty \frac{d\nu}{\nu} \left[ mW_3(\nu) + \nu W_4(\nu) \right] = - \frac{\mu_A}{4m^2},
\]

where \( \nu \) is the energy of the virtual photon in the laboratory system, \( m \) is the mass of the proton and \( \mu_A \) is its anomalous magnetic moment. SCHWINGER (1) as well as DE RAAD, MILTON and TSAI (2) have verified this sum rule within

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the context of pure electrodynamics to the lowest order in the fine-structure constant $\alpha$. They find that, in pure electrodynamics, the left-hand side of (1) comes out to be $-\alpha/8\pi\hbar^2$, so that $\mu_a = \alpha/2\pi$, which is the well-known value of the anomalous magnetic moment to lowest order in $\alpha$. The purpose of this note is to point out a simple derivation of this sum rule based on the well-known superconvergence prescription and to underline the present difficulties in saturating the left-hand side of (1). As a by-product of the discussion, we obtain the Regge behaviour of the invariant amplitudes of virtual photon-nucleon scattering, which has also been obtained by Heimann using a different method. The present method is more straightforward.

2. – Regge behaviour of virtual photon-nucleon scattering.

The problem of writing down a suitable set of invariant amplitudes for Compton scattering on polarized nucleons has recently been dealt with in great generality by Tarrach. Let $k$ and $p$ denote the four-momenta of the incoming photon and nucleon and $k'$ and $p'$ those of the outgoing photon and nucleon, respectively. The scattering amplitude is written as

$$\langle N'N | T_{\mu\nu} | pN \rangle = \varepsilon^{*\mu}(k')\bar{u}(p')T_{\mu\nu}(P, k, k')u(p)\varepsilon'(k),$$

where $P = \frac{1}{2}(p + p')$, $\varepsilon'(k)$ and $\varepsilon^{*\mu}(k')$ are the polarization vectors of the incoming and outgoing photon, respectively. We can expand $T_{\mu\nu}$ in terms of a complete set of independent tensors $I_{\mu\nu}^i(P, k, k')$:

$$T_{\mu\nu}(P, k, k') = \sum_i I_{\mu\nu}^i(P, k, k') A_i(k^2, k'^2, k\cdot k', P\cdot Q),$$

where $Q = \frac{1}{2}(k + k')$. Each of these tensors $I_{\mu\nu}^i$ is Lorentz invariant and gauge invariant and carries all the kinematics so that the invariant amplitudes $A_i$ are free from all kinematic zeros and singularities. Tarrach shows that there are 34 tensors to begin with, but two of these are dependent on the others so that one has only 32 independent tensors. The next problem of imposing gauge invariance, without at the same time introducing kinematic singularities and zeros, can be solved by the method of Bardeen and Tung, and this gives rise to 18 gauge-invariant tensors. If we now take $k^2 = k'^2$ we obtain 12 independent tensors as a consequence of time reversal invariance.

(See, for instance, V. de Alfaro, S. Fubini, G. Furlan and C. Rossetti: Currents in Hadron Physics (Amsterdam, 1973).

