Study of the Cyclic Representation of the Superlocal Scalar-Field Algebra on the Light-Cone.

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Summary. — We investigate the cyclic representation of the superlocal scalar-field algebra on the light-cone. We find that the n-point Wightman function for interacting fields can be determined completely for lightlike separation of the arguments. If the light plane commutation relations are the same for free and interacting fields, then the vacuum expectation values, restricted to a null plane, are the same.

1. - Introduction.

In this work we study the cyclic representations of the algebra of superlocal real scalar fields which are restricted to a lightlike plane. As is known from the reconstruction theorem (1) that the operators in a cyclic representation are completely determined from their vacuum expectation values (VEV), we concentrate here our attention on these basic quantities. We have taken the cyclic representation since it is probably the most interesting one. For interacting superlocal fields we have postulated the same light plane commutation relations (CR) as for the free fields. The main reason for this is to have a starting point and see what are the effects on the VEV's. We hope that the study of these representations may become relevant with respect to the scheme in which the dynamics of elementary particles must be formulated.

The main results of the present study can be summarized in a theorem which can be stated as follows: the n-point function for superlocal fields restricted to a lightlike plane can be determined completely if they are subject to the following conditions: translation invariance, spectral condition, invariance under the lightlike stability group and the postulated commutation relations for the fields. As these commutation relations are the same as for the free field, we have moreover the result that the n-point functions for free and interacting fields are exactly the same, when restricted to a lightlike plane.

2. – Notation.

In the following we will be working with light-cone co-ordinates which we denote by

\[ x^\mu = (x^0, x^1, x^2, x^3) \equiv (x^+, x^-), \quad x \equiv (x^1, x^2). \]

The ordinary co-ordinates will be denoted with a hat:

\[ \hat{x}^\mu = (\hat{x}^0, \hat{x}^1, \hat{x}^2, \hat{x}^3). \]

The relation between the two sets of co-ordinates is usually taken to be (2)

\[ x^+ = \frac{1}{\sqrt{2}} (\hat{x}^0 + \hat{x}^3), \quad x^- = \frac{1}{\sqrt{2}} (\hat{x}^0 - \hat{x}^3), \]

or

\[ x^\mu = e^\mu_\nu \hat{x}^\nu \]

with

\[ e^\mu_\nu = \begin{pmatrix} 1 & 0 & 0 & 1 \\ \sqrt{2} & 0 & 0 & \sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}. \]

For the scalar product of two 4-vectors we have

\[ x \cdot y = g_{\mu\nu} x^\mu y^\nu = x^+ y^- + x^- y^+ - x \cdot y \]