Summary. — The formulae for the amplitudes of Delbrück scattering given in a previous paper are manipulated and much simpler expressions for them are derived having the form of a sum of twofold and threefold integrals. The integrands are rather compact and contain only rational, irrational and logarithmic functions. In this preliminary work we consider only the amplitude $a_{+}$ for circularly polarized photons and we give numerical results for photon energy equal to 10.83 MeV and scattering angle in the range from 50° to 150°. For what concerns the real part of the amplitudes, these preliminary results allow one to resolve the discrepancy between the numerical results previously obtained by Ehlotzky and Sheppey and those more recently given by Papatzacos. In fact our results agree with those by Papatzacos and this seems to confirm that the fixed-angle dispersion relation used by Ehlotzky and Sheppey for the calculation of the real parts is incorrect.

1. — The amplitudes for Delbrück scattering (that is elastic scattering of a photon by nuclei) have been exactly evaluated, in the lowest perturbative order, only in the forward direction (1), as is well known. For other values of the scattering angle numerical calculations must be performed. In particular numerical results were given by EHLOTZKY and SHEPPEY (2). They used formulae for the imaginary part of the amplitudes having the form of fivefold integrals, previously derived by KESSLER (3). Moreover they obtained the real

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.


parts in the form of sixfold integrals with the aid of a fixed-angle dispersion relation. More recently Papatzacos (4), using the conventional formulation of quantum electrodynamics, derived expressions for Delbrück amplitudes in the form of a sum of threefold and fourfold integrals. However his numerical results disagree from those by Ehlotzky and Sheppey as far as the real parts of the amplitudes are concerned. A possible source of this discrepancy may be represented by the fact that the fixed-angle dispersion relation assumed by Ehlotzky and Sheppey is incorrect, as Papatzacos has pointed out. Some years ago nonlinear effects in quantum electrodynamics, with a dispersive approach, were considered in a paper by us (5). In particular we obtained the amplitudes for the Delbrück scattering in the form of threefold integrals, but no calculations were performed. In this note we give expressions much simpler than those obtained by us in A and also by other authors in previous papers, the amplitudes having the form of a sum of twofold and threefold integrals where the integrands are rather compact and contain only rational, irrational and logarithmic functions. The dilogarithm functions, which appear in a very large number of terms in the expressions derived in A, are not present now. With these more manageable formulae we have carried out numerical calculations. First of all we have directed our attention in order to try to resolve the discrepancy between the numerical results by Ehlotzky and Sheppey and those by Papatzacos. These two sets of numerical results, in particular for what concerns the real part of the amplitudes, are the only ones available in the literature on this argument, as a consequence of the complicated expressions which are involved. Therefore it is a crucial point for the comparison with the experiment to check which is the correct set. This justifies the present short note by which it results that our numerical calculations agree with those by Papatzacos. In order to exemplify, we consider only the amplitude $M_{+-}$. General expressions involving also the other amplitude $M_{++}$ and more extensive numerical calculations as well as comparison between theory and experiment shall appear in a future paper.

2. - We shall use the same notation and units as in A. In order to obtain $M_{+-}$ in a more simplified form, we start from the second of eqs. (51) of A and from the following dispersion representation for the amplitudes $f_{+-}^{(0)}$ and $f_{+-}^{(1)}$:

$$f_{+L}^{(1,2)}(1 2 3 4) = \int J_1 \, dx \, dy \left( \frac{1}{y - s - i \epsilon} + \frac{1}{y - t - i \epsilon} \right) \left( \frac{\alpha^{(1,2)}}{x - r - i \epsilon} + \beta^{(1,2)} \right) +$$

$$+ \int J_1 \, dx \, dy \left( \frac{1}{(s - m - i \epsilon)(t - n - i \epsilon)} \gamma^{(1,2)} + \delta^{(1,2)} \right),$$
