An Equivalence of Relativistic Field Equations (*) (**) .

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Summary. - A mechanism is developed whereby some relativistic wave equations for the same spin can be related to each other. The successful as well as the pathological attributes of a given equation can then be seen to extend to other theories.

1. Introduction.

Relativistic theories for particles of spin $s$ can be constructed from wave equations of the form

\begin{equation}
(-i \beta_\mu \partial^\mu + m) \psi(x) = 0,
\end{equation}

which is assumed to transform covariantly under $A \mapsto T(A)$; a reducible representation of $SL_2,\mathbb{C}$, which in turn requires that

\begin{equation}
\begin{aligned}
T \beta_\mu T^{-1} &= A_\mu^\rho \beta_\rho, \\
T \psi(x) &= \psi'(x'), \\
x'_\mu &= A_\mu^\rho x_\rho.
\end{aligned}
\end{equation}

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The mass term $m$ is assumed to be a nonzero multiple of the appropriate dimensional identity matrix (1).

It is well known that all the familiar wave equations for spins $> 1$ lead to one or more of the several unresolved difficulties in interactions. Such failures may be present in both the classical and quantum field formulations (2). However, the generality of the constructive approach to relativistic wave equations makes it evident that many different representations of the Lorentz group are available for composing theories for the same spin. One wave equation for a given spin may have a particular pathology in interaction with coupled or external fields, but another one may be able to avoid it. Different wave equations for the same spin may in general predict different physical qualities. Naturally, a question arises as to which theories for a given spin are equivalent or related to each other by some mathematical equivalence. A part of the answer will be explored in this study.

The types of equivalence relationships developed here are carried out on free-field equations, but the structure of the equivalence mechanisms is such that the equivalence extends into relationships of the equations under interactions with prescribed external fields. In this way the known physical predictions of one theory may be used to study those of another theory, without explicit calculations.

2. – The mechanism of equivalence.

This mechanisms exploits the combined properties of two types of equivalences.

1) Suppose we have a wave equation with $\beta_0$ of the form

\[
\beta_0 = \begin{bmatrix} \beta_0^{(1)} & Y_0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T^{(1)} \\ T^{(2)} \end{bmatrix}
\]

so that

\[
\left( -i \beta_\mu \gamma^\mu + m \right) \psi(x) = 0
\]

transforms under

\[
T = T^{(1)} \otimes T^{(2)}.
\]
